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FLUTTER OF A LINEAR VISCOELASTIC CANTILEVER ROD

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Annotation. This article considers the flutter problem for a linear viscoelastic cantilever rod. A statement and a method for solving the flutter problem for a linear viscoelastic cantilever rod are presented, and numerical calculations are carried out.

Keywords: viscoelasticity, rod, flutter, physical linearity, aerodynamic linearity, Bubnov-Galerkin method, relaxation kernel, numerical method, integro-differential equation.

Introduction. Accounting for the hereditary effects of deformable materials is becoming increasingly necessary due to the fact that, in most leading fields of modern engineering, various elements and components of modern engineering structures are often operated under varying conditions. The widespread use of composite materials in modern engineering has led to the need to study the optimal design of thin-walled structures with viscoelastic properties. Consequently, the hereditary theory of viscoelasticity is attracting increasing attention from researchers. This is evidenced by the publication in recent years of a number of scientific papers reflecting the latest advances in viscoelasticity theory.

This paper examines the linear flutter of a rod rigidly clamped at one end (cantilever). A one-dimensional rod model, taking into account the variability of width and thickness, allows for a more accurate representation of the rod's actual shape.

Mathematical model. Let us consider the problem of self-oscillations of a linear viscoelastic rod. We will adopt the relationship between stresses σ and strains ε in the form [1]:

$$\sigma = (1 - R^*)m_1\varepsilon, \quad \varepsilon = u_x, \; u = -zw_x \qquad (1)$$

or

$$\sigma = -(1 - R^*)m_1 z w_{xx} \qquad (2)$$

where m_1 is the elastic constant, *R(t)* - is the heredity kernel having weakly singular features of the Abel type, *E* - is the modulus of elasticity.

Without taking into account the influence of aerodynamic nonlinearity, according to the one-dimensional theory of gas, the gas pressure on the piston load is taken as [2]:

$$q = \frac{\chi p}{c} \; V\frac{\partial w}{\partial x} + \frac{\partial w}{\partial t}$$

where indicated $q = p - p$, $k = \frac{\chi p}{c}$

$$q = k \; V\frac{\partial w}{\partial x} + \frac{\partial w}{\partial t} \qquad (3)$$

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We will solve the problem of a self-oscillating process in a linear viscoelastic formulation, taking into account physical and aerodynamic linearities. To this end, we will construct a mathematical model for studying a viscoelastic rod in a gas flow, taking these linearities into account.

In this case, assuming the plane cross-section hypothesis for the bending moment, we use the following formula [6]:

$$M_x = \int_{-h/2}^{h/2} b(x)\sigma_x z dz \qquad (4)$$

(2) put into (4) and get [1]:

$$M_x = m_1(1-R^*)J_2 w_{xx} \qquad (5)$$

where are equal for beams of width $b(x)$ and height $h(x)$

$$J_2 = \frac{b(x)h^3(x)}{12}.$$

Substituting (5) into the equilibrium equation, that is,

$$\frac{\partial^2 M_x}{\partial x^2} = m(x)\frac{\partial^2 w}{\partial t^2} + q(x,t) \qquad (6)$$

and moving to dimensionless coordinates and omitting the primes, we have

$$(1-R^*)\frac{\partial}{\partial x^2}[J_2(x)w_{xx}] + F(x)w_{tt} + Pw_x + \gamma w_t = 0 \qquad (7)$$

where

$$W = h_0\overline{W},\ x = a\bar{x},\ t = t_1\bar{t},\ m = m_0F(x),\ h(x) = h_0\overline{h(x)},\ b(x) = b_0\overline{b(x)},$$

$$J_2 = J_2^0 d(x), \quad J_2(x) = b(x)h^3(x), \quad J_2^0 = \frac{b_0h_0^3}{12},$$

$$P = \frac{kVa^3}{m_1J_2^{(0)}}, \quad t_1 = \sqrt{(m_0a^4)/(m_1J_2^{(0)})}, \quad \gamma = \frac{kza^4}{m_1J_2^{(0)}t_1},\ F(x) = b(x)h(x)$$

h_0 - is the height of the rod at the ends, b_0 - is the width of the rod at the ends, and m_0 - is the mass corresponding to a unit variable cross-section of the rod.

Solution method. We construct an approximate solution using the Bubnov-Galerkin method. We represent solution (7) as

$$w = \sum_{k=1}^{N} u_k(t)\varphi_k(x) \qquad (8)$$

where $\varphi_k(x)$ - are known basis functions satisfying the given boundary conditions, and $u_k(t)$ - are unknown functions of time to be determined. N - is the number of terms in the expansion.

In this case, we obtain the following linear systems of ordinary integro-differential equations

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$$\sum_{k=1}^{N}\left[a_{ki}\ddot{u}_k(t)+\gamma b_{ki}\dot{u}_k(t)+\omega_{ki}\left(1-R^{*}\right)u_k(t)+Pd_{ki}u_k(t)\right]=0,\ i=\overline{1,N} \qquad (9)$$

where

$$a_{ki}=\int_0^1 F(x)\varphi_k(x)\varphi_i(x)dx, \qquad b_{ki}=\int_0^1 \varphi_k(x)\varphi_i(x)dx,$$

$$\omega_{ki}=\int_0^1 d(x)\varphi_k(x)\varphi_i(x)dx, \qquad d_{ki}=\int_0^1 \varphi_k(x)\varphi_i(x)dx,$$

The integration of the linear system (4) with the Rzhanitsyn–Koltunov kernel $R(t)=A\cdot e^{\beta t}t^{\alpha-1}$, $A>0$, $\beta>0$, $0<\alpha<1$ over a wide range of changes in the physical and mechanical parameters of the rod is performed using a numerical method based on analytical transformations [4]. According to this method, the numerical values of the sought functions $u_\kappa(t_l)=u_{k,l}$ are found from the solution of the following recurrent system of linear algebraic equations:

$$\sum_{k=1}^{N}\left[a_{ki}+\gamma\frac{\Delta t}{2}b_{ki}\right]u_{k,j}=\sum_{k=1}^{N}\left[(a_{ki}+\gamma t_l b_{ki})u_{k,l}+t_l\dot{u}_{k,0}\right]-\sum_{k=1}^{N}\sum_{i_1}^{l-1}\left[\gamma A_{i_1}b_{ki}u_{k,i_1}+\right.$$

$$\left.+A_{i_1}\left(t_l-t_{i_1}\right)\left(\omega_{ki}\left(u_{k,i_1}-\frac{A}{\alpha}\sum_{i_2}^{i_1}B_{i_2}e^{-\beta t_{i_2}}u_{k,i_1-i_2+1}\right)+Pd_{ki}u_{k,i_1}\right)\right],\ i=\overline{1,N} \qquad (10)$$

where

$$t_i=i\Delta t,\ B_1=\frac{\Delta t^{\alpha}}{2},\ B_{i_2}=\frac{\Delta t^{\alpha}\left[(i_2+1)^{\alpha}-(i_2-1)^{\alpha}\right]}{2},\ i_2=\overline{2,i_1-1}$$

$$B_{i_1}=\frac{\Delta t^{\alpha}\left[i_1^{\alpha}-(i_1-1)^{\alpha}\right]}{2},\ A_1=\frac{\Delta t}{2},\ A_i=\Delta t,\ i_1=\overline{2,i-1},\ i=1,2.....$$

The calculation is performed for various rheological parameters and beam planforms. The calculation is performed in both perfectly elastic and viscoelastic settings.

Beam functions are used as the basis functions $\varphi_k(x)$ [3].

$$\varphi_k(x)=\sin\lambda_k x-sh\lambda_k x-\frac{\sin\lambda_k+sh\lambda_k}{\cos\lambda_k+ch\lambda_k}(\cos\lambda_k x-ch\lambda_k x);$$

$$\lambda_1=1.875,\quad \lambda_2=4.694,\quad \lambda_3=7,855,\quad \lambda_4=10,996,\ldots,\quad \lambda_k=\frac{\pi}{2}(2k-1) \qquad (11)$$

and for the initial conditions

$$u_k(0)=\int_0^1 \alpha_0(x)\varphi_k(x)dx,\ \dot{u}_k(0)=0,\ \text{где}\ \alpha_0(x)=\{[x(1-x)]^4+\varphi_1(x)\}/100.$$

Analysis and Conclusion. Critical velocity, determined by linear theory in both perfectly elastic and viscoelastic settings, is only the upper limit of critical velocities for real structures.

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Therefore, to investigate its influence on critical velocity, it is necessary to define a specific relationship between the rod's planform shape and its stiffness and mass; that is, it is necessary to perform calculations for various α_1 and α_2. For this purpose, a series of rods with a trapezoidal planform of variable thickness are considered. The rod's shape then depends on the parameters α1 and α2, the latter parameter characterizing the taper of the rod.

The cross-section of the beam changes according to the law $b(x) = c - \alpha_1 x$, $h(x) = 1 - \alpha_2 x$, $c = 5$.

An analysis of the calculation results shows that the critical velocity in the elastic linear state is Pcr.lin = 67.69, and the difference with the viscous state is (at A = 0.05) approximately 9.2% (Pcr.lin = 61.43). Increasing the viscosity value decreases the critical velocity. When the trapezoidal wing shape is modified by parameters α_1 and α_2, the influence of the shape is significant, resulting in higher critical flutter velocity.

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