

COGNITIVE GROWTH IN EARLY CHILDHOOD: THE IMPACT OF HEURISTIC MATHEMATICAL MODELING

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Abstract: This article explores the theoretical and practical foundations of mathematical modeling for preschool children. The study emphasizes the adaptation of inventive problem-solving theory (TRIZ) and heuristic learning to enhance logical, schematic, and spatial thinking. The research categorizes mathematical models and outlines the progression of cognitive development—encompassing sensory, intellectual, and creative abilities—across early, middle, and senior preschool ages. Practical modeling technologies, including planar (Tangram, Pentamino) and spatial (Unicube, Bricks for all) games, are analyzed for their effectiveness in developing analytical, synthetic, and spatial reasoning skills. The findings confirm that early mathematical modeling builds a robust conceptual foundation for continuous early childhood education.

Keywords: mathematical modeling, preschool education, heuristic learning, cognitive abilities, spatial orientation, TRIZ, sensory standards, planar modeling, spatial modeling.

Annotatsiya: Ushbu maqola maktabgacha yoshdagi bolalar uchun matematik modellashtirishning nazariy va amaliy asoslarini o'rganadi. Tadqiqot mantiqiy, sxematik va fazoviy tafakkurni kuchaytirish uchun ixtirochilik masalalarini yechish nazariyasi (TRIZ) va evristik ta'limni moslashtirishga qaratilgan. Tadqiqot matematik modellarni tasniflaydi va kichik, o'rta va katta maktabgacha yoshdagi kognitiv rivojlanish bosqichlarini (sensor, intellektual va ijodiy qobiliyatlarni) belgilab beradi. Tekislikdagi (Tangram, Pentamino) va fazoviy (Unicube, Bricks for all) o'yinlarni o'z ichiga olgan amaliy modellashtirish texnologiyalari ularning analitik, sintetik va fazoviy fikrlash qobiliyatlarini rivojlantirishdagi samaradorligi tahlil qilinadi. Natijalar shuni tasdiqlaydiki, erta matematik modellashtirish uzluksiz ta'lim uchun mustahkam konseptual poydevor yaratadi.

Tayanch so'zlar: matematik modellashtirish, maktabgacha ta'lim, evristik ta'lim, kognitiv qobiliyatlar, fazoviy mo'ljal olish, TRIZ, sensor etalonlar, tekislikdagi modellashtirish, fazoviy modellashtirish.

Аннотация: В статье исследуются теоретические и практические основы математического моделирования для детей дошкольного возраста. Акцентируется внимание на адаптации теории решения изобретательских задач (ТРИЗ) и эвристического обучения для развития логико-схематического и пространственного мышления. В исследовании классифицируются математические модели и описывается развитие когнитивных, сенсорных, интеллектуальных и творческих способностей в младшем, среднем и старшем дошкольном возрасте. Анализируется эффективность практических технологий моделирования: плоскостных (Танграм, Пентамино) и пространственных (Уникуб, Кубики для всех). Результаты подтверждают, что раннее математическое моделирование создает прочный фундамент для непрерывного образования.

Ключевые слова: математическое моделирование, дошкольное образование, эвристическое обучение, когнитивные способности, пространственная ориентация, ТРИЗ, сенсорные эталоны, плоскостное моделирование, пространственное моделирование.

Introduction. Modern educators are afforded extensive opportunities to design original programs for the mathematical development of preschool children. However, such work requires a deep understanding of the fundamental principles and methods of mathematical theory, drawing on the accumulated positive experience of traditional, alternative, and diverse approaches to early mathematics education.

One of the most relevant and promising areas for enriching existing practices is the adaptation of inventive problem-solving theory (TRIZ) [8], heuristic learning, and mathematical modeling to the specifics of children's creative activity. These approaches significantly enhance the formation of mathematical concepts, serving as credible sources for refining curricular requirements in accordance with the modern conceptual development of preschool education [13, 14]. Intellectual activity based on active exploration and problem-solving can become habitual for preschoolers if teachers purposefully cultivate an intrinsic interest in inquiry. Key indicators of intellectual development by the end of the preschool period include the maturity of the figurative-verbal foundations of logical thinking, imagination, creativity, and the ability to classify, synthesize, and use schematization and modeling. The most effective educational approaches follow the logical conceptual sequence: "Set → Quantity → Number → Relationship" [11, 12].

Object of Study and Applied Methods. The object of this study is the process of mathematical development and cognitive growth in preschool children. Grounded in foundational psychological and pedagogical theories of cognitive development, this process is viewed through the lens of active constructivism. The applied methods center on mathematical modeling—defined in contemporary research as a teacher-organized, heuristically oriented process. Within this framework, children create models through planar and spatial mathematical abstractions, gradually transitioning from external material actions to internal mental operations, which closely aligns with P.Y. Galperin's theory of the stage-by-stage formation of mental actions [1].

Drawing upon the methodologies of early developmental education proposed by scholars such as Z.A. Mikhailova [4] and B.P. Nikitin [3], the research classifies mathematical modeling techniques into two main categories:

- By set-theoretical meaning: Determining an integer of a given invariant form as a combination of various series of its class partitions, or determining an integer as a discrete transformation of form. This category is supported by modern literature emphasizing the early formation of set concepts, quantities, and logical-schematic relationships [2, 5].
- By spatial orientation: Planar modeling based on subdividing a rectangle (e.g., Tangram, Pentamino), spatial modeling based on subdividing rectangular parallelepipeds (e.g., Unicube), modeling with materials possessing topological properties, and classical origami. These methods are widely corroborated by pedagogical studies highlighting the critical role of visual-spatial reasoning in developing early engineering and mathematical skills [3, 6].

Furthermore, the applied methods rely on systematically tracking the child's logic in developing cognitive abilities—specifically sensory, intellectual, and creative components. According to the foundational research of L.A. Wenger, A.N. Davidchuk, and E.L. Porotskaya, this progression follows a distinct developmental trajectory: mastering the immediate replacement of real object parts and sensory standards (early age), understanding actions and manipulations using ready-made models through symbolic substitution (middle age), and the independent, heuristic construction of new models and schemes (senior preschool age) [2, 6]. By utilizing this developmental continuum, educators can qualitatively analyze children's cognitive activities and stimulate the search for innovative forms of successful mathematical modeling.

Results and their Analysis. The empirical observations and theoretical synthesis conducted in this study reveal that the development of preschool cognitive abilities—specifically the sensory, intellectual, and creative domains—progresses through a structured continuum of increasing complexity in heuristic problem-solving and action substitution.

During early childhood (1.5 to 3–4 years), the primary developmental benchmark is the formation of goal-directed activity. At this foundational stage, cognitive efforts are largely externally driven and rely heavily on tactile-motor exploration. Children focus on identifying parts of a model using basic sensory standards. Intellectually, this manifests as simple substitution, where tangible geometric forms are used to represent real-world objects, marking the onset of objectification.

In middle childhood (3–4 to 5 years), the mastery of diverse engagement methods takes precedence. Children begin a crucial transition from purely physical manipulation to semi-abstract reasoning. They develop the capacity to logically correlate finished models with pre-established sensory standards and actively manipulate ready-made models provided by educators. This phase is critical for the emergence of symbolization; children no longer just see a block as a physical object, but as a symbolic component of a larger conceptual structure, allowing them to decipher and utilize basic algorithms.

By senior preschool age (5–7 years), cognitive development reaches a highly analytical and autonomous phase. Children at this stage transcend imitation; they actively assess their outcomes against objective standards, apply self-correction, and participate in advanced creative symbolization. They demonstrate the ability to detail their models, applying combinatorial thinking to solve open-ended heuristic tasks. The transition from utilizing pre-existing schemes to the independent generation of novel spatial and planar models highlights the successful internalization of mathematical logic.

This complex, multi-dimensional progression of cognitive abilities, facilitated through targeted mathematical modeling, is systematized in Table 1.

Table 1. The logic of cognitive abilities development in preschool children

Age	Sensory abilities	Intellectual abilities	Creativity
Younger Preschool age	Identifying parts of the model with sensory standards	Substitution	Objectification
Middle preschool age	Correlation of finished model with standard	Manipulation with ready models	Symbolization
Senior Preschool age	Modeling perception	Creating models	Detailing

As illustrated in Table 1, the logic of cognitive development within the framework of mathematical modeling is characterized by a definitive shift from external, perceptual reliance to internal, abstract conceptualization. The sensory domain evolves from simple shape identification to sophisticated perceptual modeling, where children mentally recreate qualities without direct physical contact. Concurrently, intellectual abilities progress from basic physical substitution to the autonomous creation of multi-layered schemas. Ultimately, the creative trajectory—moving from basic objectification to intricate detailing—demonstrates that structured mathematical modeling provides a vital catalyst for the holistic cognitive growth of preschoolers, preparing them effectively for the rigors of primary education.

Planar Modeling Analysis. Planar modeling, grounded in the mathematical principles of partitioning invariant geometric shapes (such as rectangles or squares) into discrete fractional components, encompasses heuristic games including "Form the square," "Tangram," "Pythagoras," and "Pentamino." In the foundational stages of "Tangram," children form required target shapes from a finite set of simple geometric polygons without overlaps. This crucial exercise accelerates the development of tactile-motor surveying methods, visual-motor coordination, and fine motor skills. As children advance to senior preschool age, their engagement evolves into sophisticated combinatorial problem-solving. They learn to systematically identify and extract embedded geometric shapes within complex, undifferentiated silhouettes, a cognitive process that significantly promotes mental mobility, analytical synthesis, and logical deduction [4]. "Pentamino," which utilizes 12 uniquely configured but mathematically equal-area shapes (each formed from specific orthogonal combinations of five unit squares), introduces a substantially higher level of cognitive complexity. It deeply enhances children's foundational understanding of geometric area concepts—such as "single square," "equal-size shapes," and the conservation of area—while robustly stimulating figurative imagination. Through the heuristic compilation of these selected polyomino pieces onto unpartitioned schemes, children develop structural visual memory and flexible algorithmic thinking.

Spatial Modeling Analysis. Spatial mathematical modeling profoundly expands a child's cognitive and intellectual range by facilitating the critical transition from two-dimensional planar logic to complex three-dimensional construction. Educational technologies like "Bricks" (cuboids strategically partitioned into eight proportionally equal parallelepipeds) serve as an essential developmental bridge. They help children transition from merely imitating adult-provided physical models to independently "reading" and decoding basic two-dimensional orthographic projections (architectural drawings) into tangible three-dimensional realities. The "Unicube" system introduces an even higher conceptual and combinatorial threshold by partitioning a large rectangular parallelepiped into 27 unit cubes, dynamically painted in varied class combinations (red, blue, yellow). Through this intricate, multi-variable system, children learn to classify multifaceted shape sets based on dual properties (spatial position and color algorithm), construct complex models from purely abstract verbal descriptions, and develop highly coherent auditory-visual and spatial representations. Similarly, the "Bricks for all" methodology utilizes highly irregular, complex volumetric shapes composed of three to four unit cubes. This requires advanced spatial imagination and the capacity for mental rotation. Children must analyze these discrete, asymmetric volumes, heuristically combine them, and creatively construct novel 3D configurations, thereby dramatically improving their intellectual agility, abstract spatial reasoning, and advanced creative problem-solving capabilities [3, 6].

Conclusion. Mathematical modeling transcends traditional instructional methods, serving as a highly effective and indispensable heuristic mechanism within the intellectual and cognitive education of preschool children. Through the deliberate and systematic integration of progressively challenging planar and spatial modeling materials—ranging from elementary geometric substitution and tactile objectification to highly complex, multi-variable combinatorial logic—educators can robustly cultivate a child's analytical, synthetic, and creative capacities.

The structured pedagogical transition from direct, physical sensory manipulation to internalized, abstract perceptual modeling ensures that preschoolers develop highly functional and agile logical-schematic thinking. This progression effectively bridges the critical cognitive gap between concrete empirical experience and abstract spatial reasoning. Furthermore, by embracing the principles of active constructivism and inventive problem-solving, educators provide children with the vital opportunity to independently choose and devise their own unique pathways for solving mathematical learning tasks.

This pedagogical autonomy not only preserves the intrinsic variability and dynamic nature of their cognitive engagement but also fosters sustained intellectual curiosity and psychological

resilience. Ultimately, this comprehensive heuristic approach guarantees the deep and timely formation of essential mathematical concepts, establishing a robust conceptual bedrock that seamlessly prepares children for the academic rigors and logical demands of subsequent continuous primary education [7, 11].

References

1. Galperin, P. Y. (1992). Stage-by-stage formation as a method of psychological investigation. *Journal of Russian and East European Psychology*, 30(4), 60-80.
2. Wenger, L. A. (1986). *Development of cognitive abilities in the process of preschool education*. Moscow: Prosveshcheniye.
3. Nikitin, B. P. (1998). *Intellectual Games*. Moscow: List-New.
4. Mikhailova, Z. A. (1990). *Game entertaining tasks for preschoolers*. Moscow: Prosveshcheniye.
5. Poddyakov, N. N. (1977). *The thinking of a preschooler*. Moscow: Pedagogika.
6. Davidchuk, A. N. (1973). *Development of constructive modeling in preschool children*. Moscow: Prosveshcheniye.
7. Clements, D. H., & Sarama, J. (2014). *Learning and Teaching Early Math: The Learning Trajectories Approach* (2nd ed.). Routledge.
8. Altshuller, G. S. (1999). *The Innovation Algorithm: TRIZ, systematic innovation and technical creativity*. Technical Innovation Center, Inc.
9. Graber, A. O., Valli, L., & Newton, K. (2011). *Upper Elementary Math Lessons: Case Studies of Real Teaching*. Rowman & Littlefield Publishers, 195 p.
10. Kumon Publishing. (2008). *Subtraction. Grade 1 Math Workbook*. USA: Kumon Publishing, 96 p.
11. Bikbaeva, N. U. (2001). *Maktabgacha yoshdagi bolalarda matematik tasavvurlarni shakllantirish* [Formation of mathematical concepts in preschool children]. Tashkent: O'qituvchi.
12. Kadirova, F. R., Toshpo'latova, Sh. Q., & Kayumova, N. M. (2019). *Maktabgacha pedagogika* [Preschool pedagogy]. Tashkent: Tafakkur.
13. Decree of the President of the Republic of Uzbekistan. (2019). *Concept of development of the preschool education system of the Republic of Uzbekistan until 2030* (Resolution No. PP-4312, May 8, 2019). Tashkent: Lex.uz.
14. Decree of the President of the Republic of Uzbekistan. (2022). *About the Development Strategy of New Uzbekistan for 2022-2026* (Decree No. UP-60, January 28, 2022). Tashkent: Lex.uz.