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METHODOLOGICAL FOUNDATIONS FOR DEVELOPING A SYSTEM OF ALGEBRAIC PROBLEMS BASED ON PROBLEM-BASED LEARNING

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Abstract

This article examines the issues of developing students' logical, analytical, and creative thinking skills through the application of problem-based learning technology in modern mathematics lessons. The methodology of designing a system of algebraic problems based on the principles of consistency and systematicity is demonstrated. Furthermore, using a problem-based task related to probability theory as an example, the mechanisms of guiding students to avoid ready-made algorithms and engage in independent research are analyzed.

Keywords: problem-based learning, algebraic problems, cognitive dissonance, hypothesis, conditional probability, Bertrand's paradox, didactics.

Annotatsiya Ushbu maqolada zamonaviy matematika darslarida muammoli ta'lim texnologiyasini qo'llash orqali o'quvchilarning mantiqiy, tahliliy va kreativ fikrlash qobiliyatini rivojlantirish masalalari tadqiq etilgan. Algebraik masalalar tizimini izchillik va tizimlilik prinsiplari asosida loyihalash metodikasi ko'rsatib o'tilgan. Shuningdek, ehtimollar nazariyasiga oid muammoli topshiriq misolida o'quvchilarni tayyor algoritmlardan qochishga va mustaqil izlanishga yo'naltirish mexanizmlari tahlil qilingan.

Kalit so'zlar: muammoli ta'lim, algebraik masalalar, kognitiv dissonans, gipoteza, shartli ehtimollik, Bertran paradoksi, didaktika.

INTRODUCTION Today, the integration processes taking place within the global educational space require preparing students not merely to receive ready-made knowledge, but to think independently and make timely, correct decisions in problematic situations. Mathematics, and the algebra course in particular, often poses difficulties for students due to its high level of abstraction and its reliance on strict laws. In traditional lessons, memorizing formulas and mechanically solving template (shablon) tasks predominate, which ultimately limits a student's mathematical potential.

Problem-based learning, on the other hand, transforms the student into an active subject of the lesson, establishing autonomous knowledge-acquisition skills. By structuring a system of algebraic problems around problematic scenarios, we create an intentional "intellectual difficulty"—or **cognitive dissonance**—in the student's mind. This encourages them to understand the topic more deeply and guides them to logically discover mathematical patterns on their own, rather than simply memorizing them.

1. Didactic Principles of Problem-Based Learning

Problem-based learning is a systematically organized educational process where the teacher intentionally introduces problem situations and guides the active, exploratory activities of the students to resolve them. As the prominent pedagogue M.I. Makhmutov emphasized in his fundamental research, the core essence of problem-based learning lies in the moment a student encounters a contradiction between their existing knowledge and new requirements while completing a task, thereby forcing them to search for a new method of action [1, p. 245].

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In algebra lessons, a system of problem-oriented tasks should not be just a random collection of difficult exercises; rather, it must follow a specific didactic hierarchy:

1. **Motivational Stage:** Creating a situation where an old, familiar algorithm known to the student no longer works.
2. **Exploratory Stage:** Breaking down the problem into smaller parts, constructing a mathematical model, and putting forward a hypothesis (an educated guess).
3. **Generalization Stage:** Deriving a new formula, theorem, or algebraic method, and drawing a final conclusion.

The structural difference between traditional and problem-based approaches when solving algebraic problems is illustrated in the comparative table below:

Evaluation Criteria	**Traditional Approach**	**Problem-Based Approach**
Primary Goal of the Task	Reinforcing formulas and mechanical drilling	Searching for new knowledge and discovering patterns
Student's Role in Class	An executor working by a ready-made template	A researcher formulating hypotheses independently
Attitude Toward Mistakes	A negative outcome that lowers the grade	A natural part of exploration and a source of learning

2. Application of a Problem-Based Task Using Probability Theory

In topics where probability theory and algebraic modeling intersect, students frequently apply combinatorics formulas mechanically without deep analysis. To break stereotypical template thinking and create a genuine problem situation, it is highly effective to utilize the following classic problem (a simplified version of Bertrand's box paradox):

Problem: A box contains 3 envelopes. The first envelope contains two red cards, the second contains two blue cards, and the third contains one red and one blue card. A single envelope is chosen at random, and one card is drawn from it—it turns out to be red. What is the probability that the second card remaining inside this envelope is also red?

Emergence of the Problem Situation and Hypothesis Analysis:

Stereotypical Thinking (The Incorrect Approach): Students usually reason as follows: "Since a red card was drawn, this must either be the envelope with two red cards or the envelope with one red and one blue card. That leaves exactly two options. For the second card to be red, we need the first envelope. Therefore, the probability is $1/2$ (or 50%)".

The Problem: While this answer seems completely logical and intuitive at first glance, it is mathematically **incorrect**. A cognitive contradiction arises here between the elementary definition of basic probability and the concept of conditional probability.

Algebraic and Logical Solution (Exploratory Stage):

During the analytical discussion, the teacher guides students to count individual **cards** as separate elements rather than counting the envelopes as Wholes. In total, there are 3 red cards across the boxes, which we can index as follows:

1. R_1 (The first red card in Envelope 1)
2. R_2 (The second red card in Envelope 1)
3. R_3(The single red card in Envelope 3)

The first red card we draw and observe is guaranteed to be one of these three options. Now, let us test the condition for each possible case:

If the observed card is R_1, the remaining card in that envelope is R_2 (Red). (Condition satisfied)

If the observed card is R_2, the remaining card in that envelope is R_1 (Red). (Condition satisfied)

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If the observed card is \$R_3\$, the remaining card in that envelope is blue.
(Condition not satisfied)

Out of a total of 3 equally likely possibilities (n=3), exactly 2 cases (m=2) satisfy our required condition. According to the classical algebraic formula for probability ($P=\frac{n}{m}$ $P=\frac{2}{3}$ approx 66.7\%)

By introducing this problem into the classroom, the instructor allows students to deeply experience conditional probability through everyday logic and problem-solving without initially heavy, dry formulas. As George Pólya pointed out, such minor discoveries during the problem-solving process ignite an innate investigative drive within the student [2, pp. 41-44].

CONCLUSION A system of algebra tasks structured around problem-based learning principles serves as one of the most effective tools for increasing students' cognitive engagement. As demonstrated through the example of probability and algebraic modeling, problem-based assignments steer students away from superficial, template-driven thinking and build the capacity to resolve logical contradictions. Consistently applying this method significantly improves lesson efficiency and markedly drives student interest in competitive mathematics and science olympiads.

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