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THE FORMATION OF DIFFERENTIAL AND INTEGRAL CALCULUS BASED ON THE WORKS OF NEWTON AND LEIBNIZ

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Abstract

This article examines the historical and mathematical formation of differential and integral calculus on the basis of the scientific works of Isaac Newton and Gottfried Wilhelm Leibniz. The study analyzes the idea of infinitesimals, the problems of tangents and quadratures, the need to express motion and rates of change mathematically, Newton's method of fluxions, and Leibniz's symbolic system of differentials and integrals. It is argued that calculus did not emerge suddenly, but was the logical continuation of earlier mathematical ideas developed by Cavalieri, Fermat, Descartes, Barrow, Wallis, and other scholars. Newton interpreted calculus mainly through mechanical motion and time-dependent quantities, whereas Leibniz transformed it into a general symbolic and algorithmic method. The article also discusses the Newton–Leibniz priority dispute, the differences between their approaches, and the methodological importance of studying this historical process in modern higher mathematics education. The practical section contains two solved examples illustrating differentiation and integration in the context of the historical development of calculus.

Keywords: differential calculus, integral calculus, Newton, Leibniz, fluxion, differential, infinitesimals, quadrature, tangent, history of mathematics.

1. Introduction

Differential and integral calculus is one of the central branches of modern mathematics. It provides the mathematical language for describing rates of change, tangents to curves, motion of bodies, areas, volumes, work, energy, probability distributions, economic growth, and numerous natural and technical processes. For this reason, the emergence of calculus should not be regarded merely as an event within mathematics itself, but as one of the major intellectual transformations of seventeenth-century science [7].

The formation of differential and integral calculus is traditionally associated with the names of Isaac Newton and Gottfried Wilhelm Leibniz. This association is well justified: Newton explained changing quantities by means of his method of fluxions, whereas Leibniz introduced the notation of differentials and integrals and turned calculus into a convenient symbolic system [3], [4]. However, their achievements did not arise in isolation. Before them, Cavalieri's method of indivisibles, Fermat's and Descartes' work on tangents, Barrow's insight into the relationship between differentiation and integration, and Wallis's studies of infinite series created an important theoretical foundation [1], [8].

By the seventeenth century, mathematics faced two major problems. The first was the problem of finding a tangent to a curve at an arbitrary point, which is equivalent in modern terms to determining a rate of change. The second was the problem of quadrature, that is, finding the area under a curve. In modern language, the first problem led to differentiation, while the second led to integration. The great achievement of Newton and Leibniz was that they understood the deep internal connection between these two problems and raised this connection to the level of a general mathematical method [5], [6].

The purpose of this article is to analyze the formation of differential and integral calculus on the basis of the works of Newton and Leibniz from a historical and mathematical point of

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view, to identify the similarities and differences between their methods, and to demonstrate the importance of this topic in modern higher mathematics education.

The main objectives of the article are: to examine the mathematical background that existed before the creation of calculus; to explain Newton's method of fluxions; to present Leibniz's system of differential and integral notation; to discuss the scientific consequences of the Newton–Leibniz priority dispute; and to support the theoretical ideas through two solved examples.

2. Research Methods

The article uses historical-mathematical analysis, comparison, theoretical generalization, and mathematical modeling. As a principal historical source, G. G. Zeuthen's work devoted to the history of mathematics in the sixteenth and seventeenth centuries is used. In this work, the method of infinitesimals, ideas of integration, the tangent problem, and the development of the methods of Newton and Leibniz are discussed in historical context [1].

In addition, Newton's *Philosophiae Naturalis Principia Mathematica* and *The Method of Fluxions and Infinite Series*, as well as Leibniz's 1684 paper *Nova methodus pro maximis et minimis...* and his 1686 paper on integral calculus, are considered as fundamental theoretical sources [2], [3], [4], [5]. In the mathematical analysis, modern notation is used because the notation found in historical sources differs significantly from the notation used today.

The research was carried out in the following directions: identifying the mathematical conditions that preceded differential and integral calculus; comparing the main concepts in the approaches of Newton and Leibniz; explaining derivative and integral concepts through historical problems; showing the modern interpretation of their methods through solved examples; and drawing methodological conclusions for higher mathematics education.

3. Scientific Background Before Differential and Integral Calculus

3.1. Quadrature and the Idea of Indivisibles

The historical roots of integral calculus go back to problems of finding areas and volumes. Ancient Greek mathematicians, especially Archimedes, used highly refined limiting methods to determine the area of a parabolic segment and the volume of a sphere. However, these methods were not formulated as a general algorithm; rather, they appeared as separate geometric proofs [8].

In the seventeenth century, Bonaventura Cavalieri introduced the method of indivisibles. According to this idea, a plane figure could be regarded as consisting of infinitely many line segments, and a solid body as consisting of infinitely many plane sections. This idea was one of the earliest intuitive forms of what later became the concept of the integral [1], [8]. For example, imagining the area under a curve as the sum of a very large number of thin rectangles is a natural idea that leads to the modern Riemann integral.

The importance of Cavalieri's idea lies not only in its computational value, but also in its conceptual influence. It allowed mathematicians to think of continuous geometric objects as being composed of infinitesimal elements. Although this was not yet a rigorous theory in the modern sense, it created a powerful intuitive foundation for the later development of integral calculus.

3.2. The Tangent Problem and the Path Toward Differentiation

The second major source of differential calculus was the tangent problem. Descartes and Fermat developed different methods for drawing tangents to curves. Fermat, in particular, came close to the idea of using very small changes in a quantity when solving maximum and minimum problems. His approach is closely related to the modern method of finding extrema by setting the derivative equal to zero [1], [7].

The tangent problem was not merely a geometric question. If a curve is given by the equation $y = f(x)$, the slope of the tangent at the point $x = a$ is expressed in modern notation by the limit

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$$f\ (a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}.$$

Of course, in the early seventeenth century the concept of a limit did not yet exist in the rigorous form used today. Nevertheless, mathematicians were practically investigating what value such a ratio approached when h became extremely small. Thus, before the strict theory of limits was developed, the intuitive idea of infinitesimals was already functioning in the history of differential calculus [7], [8].

3.3. Barrow and the Idea of Inverse Operations

Isaac Barrow occupies an important intermediate position in the formation of calculus. He recognized a connection between the problem of finding tangents and the problem of finding areas. In modern language, if $F'(x) = f(x)$, then

$$_a^b\, f(x), dx = F(b) - F(a).$$

This relation expresses the content of the fundamental theorem of calculus. Although Barrow did not develop it as a complete algebraic algorithm, his views created an important theoretical foundation for Newton and Leibniz [6], [8].

Barrow's contribution is especially significant because it connected two problems that had previously been treated largely separately: the local problem of tangency and the global problem of area. This connection later became the core of mathematical analysis.

4. Newton's Method of Fluxions

Newton explained calculus mainly through motion, time, and variable quantities. He called a quantity that changes with time a "fluent," and the rate of change of that quantity a "fluxion" [3]. If (x) is a time-dependent variable, its fluxion is denoted in modern notation by $\dot{x}$, that is,

$$\dot{x} = \frac{dx}{dt}.$$

Newton's approach had a mechanical and kinematic character because he aimed to express physical motion mathematically. For example, if the distance traveled by a body is given by $s = s(t)$, then its velocity is

$$v = \frac{ds}{dt},$$

and its acceleration is

$$a = \frac{d^2 s}{dt^2}.$$

These ideas later became part of the mathematical foundation of classical mechanics [2], [9]. Newton's calculus was therefore not only a symbolic technique, but also a powerful method for describing natural laws quantitatively.

Another important aspect of Newton's work was his extensive use of infinite series and the binomial expansion. For example, the generalized binomial series

$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!}x^3 + \cdots$$

allowed Newton to represent complicated functions by means of infinite series [3], [7]. This played an important role in the further development of calculus, particularly in approximation methods, astronomical calculations, and mechanical modeling.

Although Newton's *Principia* did not present calculus in the same explicit symbolic form as modern textbooks, many of its results were based on limiting ratios and rates of change [2]. Newton preserved a geometric style while giving mathematical form to the laws of motion. Therefore, his contribution to calculus should be evaluated not only as an algebraic method, but also as a methodology for the mathematical description of nature [9].

5. Leibniz's System of Differential and Integral Calculus

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Leibniz's approach differed from Newton's in that it was more symbolic, algorithmic, and general. He introduced such notations as dx, dy, $\frac{dy}{dx}$, and , thereby creating a convenient and consistent system of calculation [4], [5]. The notation $\frac{dy}{dx}$, in particular, represents the derivative as a ratio of differentials and makes many operations intuitive as well as formally manageable.

In his 1684 paper, Leibniz presented a new method for solving problems of maxima, minima, and tangents [4]. He formulated rules for calculating differentials. For example, if

$y = uv$,

then

$dy = u,dv + v,du$.

This corresponds to the modern product rule

$(uv)' = u'v + uv'$.

Similarly, if

$$y = \frac{u}{v},$$

then

$$dy = \frac{v,du - u,dv}{v^2},$$

which corresponds to the modern quotient rule.

Leibniz's 1686 paper is especially important for integral calculus. In it, the integral sign was used, and integration was connected with the idea of summation [5]. The symbol itself is commonly understood as a stretched form of the letter S, derived from the Latin word *summa*. This is conceptually meaningful because an integral can be interpreted as the limiting sum of infinitely many small quantities.

The advantage of Leibniz's notation was that it was quickly adopted by later mathematicians, especially Jacob and Johann Bernoulli, Euler, Lagrange, and others. As a result, mathematical analysis developed rapidly on the European continent [7], [10]. Even today, differential and integral calculus mainly uses Leibniz's notation:

$$\frac{dy}{dx}, \qquad {}_a^b f(x),dx, \qquad d^2y, \qquad \frac{\partial z}{\partial x}.$$

This shows that Leibniz's symbolic system was extremely convenient from both scientific and methodological points of view [6], [8].

6. Comparative Analysis of Newton's and Leibniz's Approaches

Newton and Leibniz are generally considered to have developed the main ideas of differential and integral calculus independently. Historical literature often emphasizes that Newton developed many of the basic ideas earlier, while Leibniz published his results earlier [7], [10]. This situation later led to the famous priority dispute.

The strength of Newton's method lay in its physical meaning. His method of fluxions naturally expressed motion, velocity, and acceleration. This made it highly effective for mechanics, astronomy, and the natural sciences. For this reason, Newton's mathematical analysis developed in close connection with classical mechanics [2], [9].

The strength of Leibniz's method lay in its universal symbolic system. The introduction of dx, dy, $\frac{dy}{dx}$, and standardized calculation. These symbols were convenient for working with complicated functions, constructing differential equations, analyzing functions of several variables, and generalizing integral formulas [4], [5].

The difference between the two approaches can be summarized as follows:

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Aspect	Newton's Approach	Leibniz's Approach
Main concept	Fluent and fluxion	Differential and integral
Notation	$\dot{x}$, $\dot{y}$	dx, dy, $\frac{dy}{dx}$,
Main interpretation	Motion and change with respect to time	Infinitesimal increments and symbolic algorithm
Strong side	Physical meaning and connection with mechanics	General rules of calculation and convenient notation
Main influence	English mathematical school and mechanics	Continental European mathematical school

The Newton–Leibniz dispute also had certain negative consequences. English mathematicians remained attached to Newtonian notation for a long time, whereas continental European mathematicians used Leibnizian notation. As a result, in the early eighteenth century, English mathematics lagged behind continental mathematics in some areas, partly because Leibniz's notation was more effective in complex calculations [10].

Nevertheless, from a modern historical point of view, both Newton and Leibniz are recognized as founders of mathematical analysis. Their independent but closely related ideas show that the emergence of differential and integral calculus was a necessary result of the scientific development of the seventeenth century [7], [8].

7. The Fundamental Theorem of Differential and Integral Calculus

The most important idea that united differential and integral calculus into a single theory is that differentiation and integration are inverse operations. If $F(x)$ is an antiderivative of $f(x)$, that is,

$$F'(x) = f(x),$$

then

$${}_a^b f(x),dx = F(b) - F(a).$$

This formula is known today as the fundamental theorem of calculus. Its historical roots go back to the works of Barrow, Newton, and Leibniz [5], [6].

The meaning of this theorem is that the tangent problem and the area problem are not independent of one another. The derivative expresses a local rate of change, whereas the integral expresses the accumulation of these changes. For example, if velocity $v(t)$ is known, then the distance traveled by a body over the time interval $[a,b]$ is

$$s = {}_a^b v(t),dt.$$

Conversely, if the position function $s(t)$ is known, then the velocity is determined by

$$v(t) = s'(t).$$

This relationship is a central idea both in Newtonian mechanics and in modern mathematical analysis [2], [9].

8. Practical Examples

Example 1. Finding a Derivative by Newton's Method of Fluxions

Consider the function

$$y = x^3.$$

In Newton's interpretation, x and y are fluents, that is, quantities changing with time. Their fluxions are denoted by $\dot{x}$ and $\dot{y}$. In modern notation, this means differentiating with respect to time:

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$$\frac{dy}{dt} = \frac{d}{dt}(x^3).$$

Using the chain rule, we obtain

$$\frac{dy}{dt} = 3x^2 \frac{dx}{dt}.$$

In Newton's notation this is written as

$$\dot{y} = 3x^2 \dot{x}.$$

If $\dot{x} = 1$, then

$$\dot{y} = 3x^2.$$

Therefore, the derivative of $y = x^3$ with respect to x is

$$y \; = 3x^2.$$

This example shows the direct connection between Newton's method of fluxions and the modern concept of the derivative.

Example 2. Using the Fundamental Theorem of Calculus to Compute an Integral

Compute the definite integral

$${}_{1}^{3} 2x, dx.$$

First, we find an antiderivative of the function $2x$. Since

$$\frac{d}{dx}(x^2) = 2x,$$

we have

$$F(x) = x^2.$$

By the fundamental theorem of calculus,

$${}_{1}^{3} 2x, dx = F(3) - F(1).$$

Now we calculate

$$F(3) = 3^2 = 9,$$

and

$$F(1) = 1^2 = 1.$$

Therefore,

$${}_{1}^{3} 2x, dx = 9 - 1 = 8.$$

Geometrically, this result represents the area under the line $y = 2x$ on the interval $[1,3]$. This example illustrates how the integral, historically connected with the quadrature problem, is calculated through the antiderivative.

9. Results and Discussion

The analysis shows that differential and integral calculus emerged in the seventeenth century as a result of the interaction between mathematical needs, mechanical problems, and the development of symbolic calculation. Although Newton and Leibniz stood at the center of this process, their works developed in continuity with earlier mathematical traditions [1], [8].

Newton's approach was strong because of its physical meaning. He considered variable quantities as depending on time and used fluxions to express velocity, acceleration, and other rates of change. This method was extremely effective for mechanics and astronomy [2], [3]. Leibniz, on the other hand, introduced differential and integral symbols and transformed calculus into a general algorithmic method. It was precisely this symbolic system that was widely accepted by later mathematicians [4], [5].

The solved examples in this article demonstrate that historical problems still have significant methodological value in modern courses of higher mathematics. When the derivative is explained through tangent slope and rate of change, and the integral through area and accumulation, students understand the topic more deeply. Moreover, comparing the methods of Newton and Leibniz helps students see how mathematical symbols, concepts, and methods are formed [6], [9].

The Newton–Leibniz priority dispute is historically complex. However, from a scientific point of view, the most important conclusion is that each of them made an independent and original contribution to mathematical analysis. Newton deepened calculus as a tool for describing mechanical processes, whereas Leibniz turned it into a universal algebraic language [7], [10].

10. Methodological Importance in Higher Mathematics Education

Studying the history of differential and integral calculus through the works of Newton and Leibniz is important in higher mathematics education for several reasons.

First, the historical approach prevents students from memorizing formulas mechanically. A student understands the derivative not only as a rule, but also as an answer to problems of tangency, velocity, and rate of change. Similarly, the integral is understood not merely as a computational operation, but as a mathematical expression of summation, area, and accumulation.

Second, comparing the methods of Newton and Leibniz shows the importance of mathematical notation. The same mathematical idea can develop differently depending on how it is symbolically expressed. The convenience of Leibniz’s notation proves the significant role that notation plays in the development of mathematical thinking [6], [8].

Third, this topic strengthens interdisciplinary connections. Newton’s method is closely related to mechanics, whereas Leibniz’s method is connected with algebra, logic, and general symbolic calculation. This makes it possible to teach mathematical analysis in connection with physics, engineering, economics, and computer science.

Fourth, the historical material is highly suitable for problem-based learning. A teacher may ask questions such as: Why does the tangent problem lead to the derivative? Why is the area problem solved by means of the integral? Why did Leibniz’s notation become dominant in the history of calculus? What is the relationship between Newton’s fluxion and the modern derivative? Such questions encourage students to think independently and actively.

Conclusion

The formation of differential and integral calculus is one of the greatest achievements of seventeenth-century mathematics. Although this process resulted from the work of many scholars over several centuries, Newton and Leibniz were the principal figures who raised calculus to the level of a general theory.

Newton analyzed the change of variable quantities with respect to time through his method of fluxions. His approach became a powerful tool for the mathematical description of mechanics, astronomy, and natural science. Leibniz introduced differential and integral notation and transformed calculus into a systematic, convenient, and widely applicable algorithmic method.

The fundamental theorem of calculus united these two directions into a single theory. The derivative expresses local rate of change, while the integral expresses the accumulation of such changes. For this reason, mathematical analysis became a universal language of modern science and technology.

The analysis and examples presented in this article show that studying the legacy of Newton and Leibniz from a historical and mathematical perspective has great methodological importance in higher mathematics education. This approach increases students’ interest in the subject, deepens their understanding of mathematical concepts, and develops scientific thinking.

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References:

1. Zeuthen, H. G. History of Mathematics in the Sixteenth and Seventeenth Centuries. Translated from German by P. Novikov; edited, annotated, and introduced by M. Vygodsky. Moscow–Leningrad: ONTI, 1938.
2. Newton, I. Philosophiae Naturalis Principia Mathematica. Londini: Jussu Societatis Regiae ac Typis Josephi Streater, 1687.
3. Newton, I. The Method of Fluxions and Infinite Series; with its Application to the Geometry of Curve-lines. Translated from the Latin by John Colson. London: Henry Woodfall; sold by John Nourse, 1736.
4. Leibniz, G. W. Nova methodus pro maximis et minimis, itemque tangentibus, quae nec fractas nec irrationales quantitates moratur, et singulare pro illis calculi genus. *Acta Eruditorum*. Lipsiae, October 1684, pp. 467–473.
5. Leibniz, G. W. De geometria recondita et analysi indivisibilium atque infinitorum. *Acta Eruditorum*. Lipsiae, June 1686, pp. 292–300.
6. Struik, D. J. A Source Book in Mathematics, 1200–1800. Princeton: Princeton University Press, 1986.
7. Boyer, C. B. The History of the Calculus and Its Conceptual Development. New York: Dover Publications, 1959.
8. Edwards, C. H. The Historical Development of the Calculus. New York: Springer-Verlag, 1979.
9. Katz, V. J. A History of Mathematics: An Introduction. 3rd ed. Boston: Addison-Wesley, 2009.
10. Hall, A. R. Philosophers at War: The Quarrel between Newton and Leibniz. Cambridge: Cambridge University Press, 1980.