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CHARGED SPHERICALLY SYMMETRIC BLACK HOLE SOLUTION WITH A - PARAMETER

Urinov Sunnatillo
Institute of Astronomy, Academy of Sciences of the Republic of Uzbekistan, Uzbekistan
mansanovich@mail.ru +998997764117

Abstract

In this work, we investigate a static and spherically symmetric charged black hole solution in the presence of a dimensionless parameter . Starting from the Einstein-Maxwell field equations, we derive an exact analytical form of the metric function. The horizon structure is analyzed in detail by solving the corresponding cubic equation. Thermodynamic quantities such as surface gravity and Hawking temperature are also discussed. The obtained solution reduces to the standard Reissner-Nordström black hole in the appropriate limit.

INTRODUCTION

Black hole solutions play a fundamental role in understanding the structure of spacetime in general relativity and its extensions. Modifications of classical solutions often introduce additional parameters that encode deviations from Einstein gravity or effective matter couplings. In this paper, we consider a charged and static spherically symmetric spacetime characterized by an additional dimensionless parameter. Such a parameter may arise in effective gravitational theories or phenomenological modifications of general relativity.

SPACETIME METRIC

We consider a static and spherically symmetric spacetime described by the line element

$$ds^2=-F(r)\,dt^2+\frac{dr^2}{F(r)}+r^2\left(d\theta^2+\sin^2\theta\, d\phi^2\right) \quad (1)$$

where $F(r)$ is the metric function to be determined.
The Einstein field equations are given by

$$G_{\mu\nu}=8\pi T_{\mu\nu}, \quad (2)$$

where the energy-momentum tensor of the electromagnetic field is

$$T_{\mu\nu}=\frac{1}{4\pi}\left(F_{\mu\alpha}F_{\nu}^{\alpha}-\frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right). \quad (3)$$

The Maxwell equations,

$$\nabla_{\mu}F^{\mu\nu}=0 \quad (4)$$

lead, under spherical symmetry, to the non-vanishing component

$$F_{tr}=\frac{Q}{r^2} \quad (5)$$

where Q is the electric charge.

EXACT BLACK HOLE SOLUTION

Substituting the metric ansatz into the Einstein equations, the (t,t) and (r,r) components yield

$$\frac{d}{dr}(rF(r))=\frac{1}{1-\gamma}-\frac{Q^2}{(1-\gamma)r^2} \quad (6)$$

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Integrating, we obtain

$$rF(r)=\frac{r^2}{2(1-\gamma)}+\frac{Q^2}{(1-\gamma)r}-2M, \tag{7}$$

where M is an integration constant identified as the black hole mass.
Therefore, the metric function reads

$$F(r)=\frac{r}{1-\gamma}-\frac{2M}{r}+\frac{Q^2}{(1-\gamma)r^2}.$$

(8)

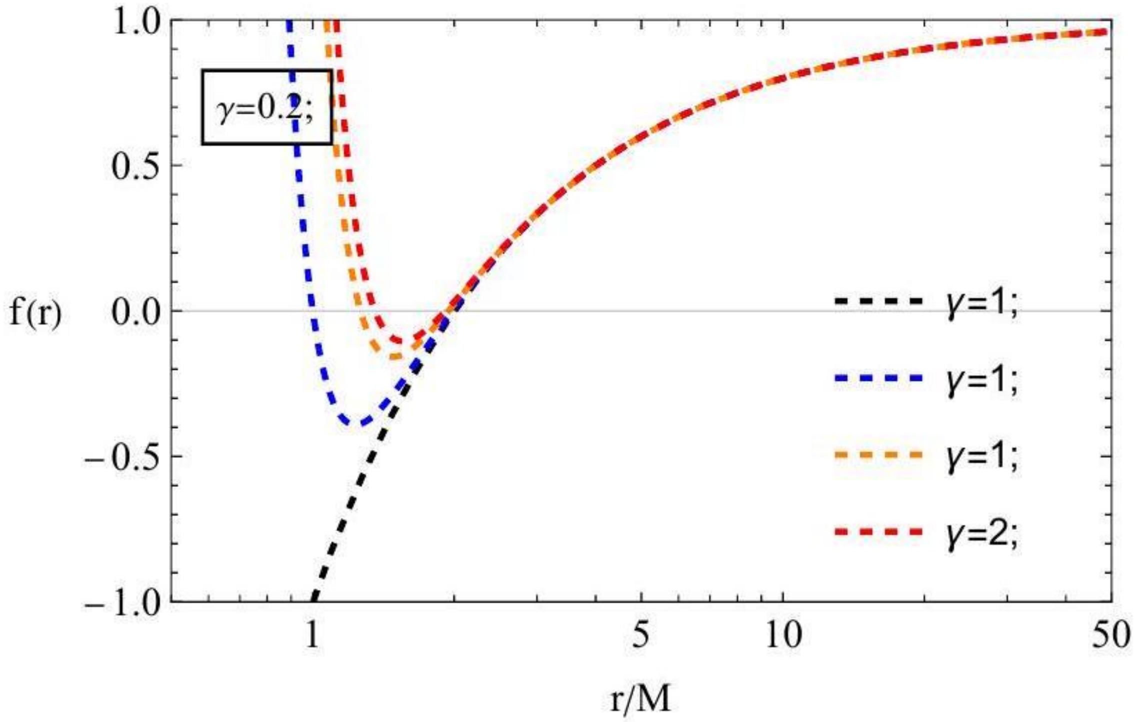

FIG. 1. Graph describing the behavior of the function $f(r)$, at different discrete values of the parameters and

HORIZON STRUCTURE

The event horizons are determined by the condition

$$F(r)=0, \tag{9}$$

which leads to the cubic equation

$$r^3-2M(1-\gamma)r+Q^2=0. \tag{10}$$

This equation can be written in the standard form

$$r^3+pr+q=0 \tag{11}$$

with

$$p = -2M(1-\gamma),\ q = Q^2. \tag{12}$$

The discriminant is

$$\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3 = \frac{Q^4}{4} - \frac{8M^3(1-\gamma)^3}{27} \tag{13}$$

For $\Delta > 0$, the event horizon radius is given by

$$r_h = \sqrt[3]{-\frac{Q^2}{2} + \sqrt{\Delta}} + \sqrt[3]{-\frac{Q^2}{2} - \sqrt{\Delta}} \tag{14}$$

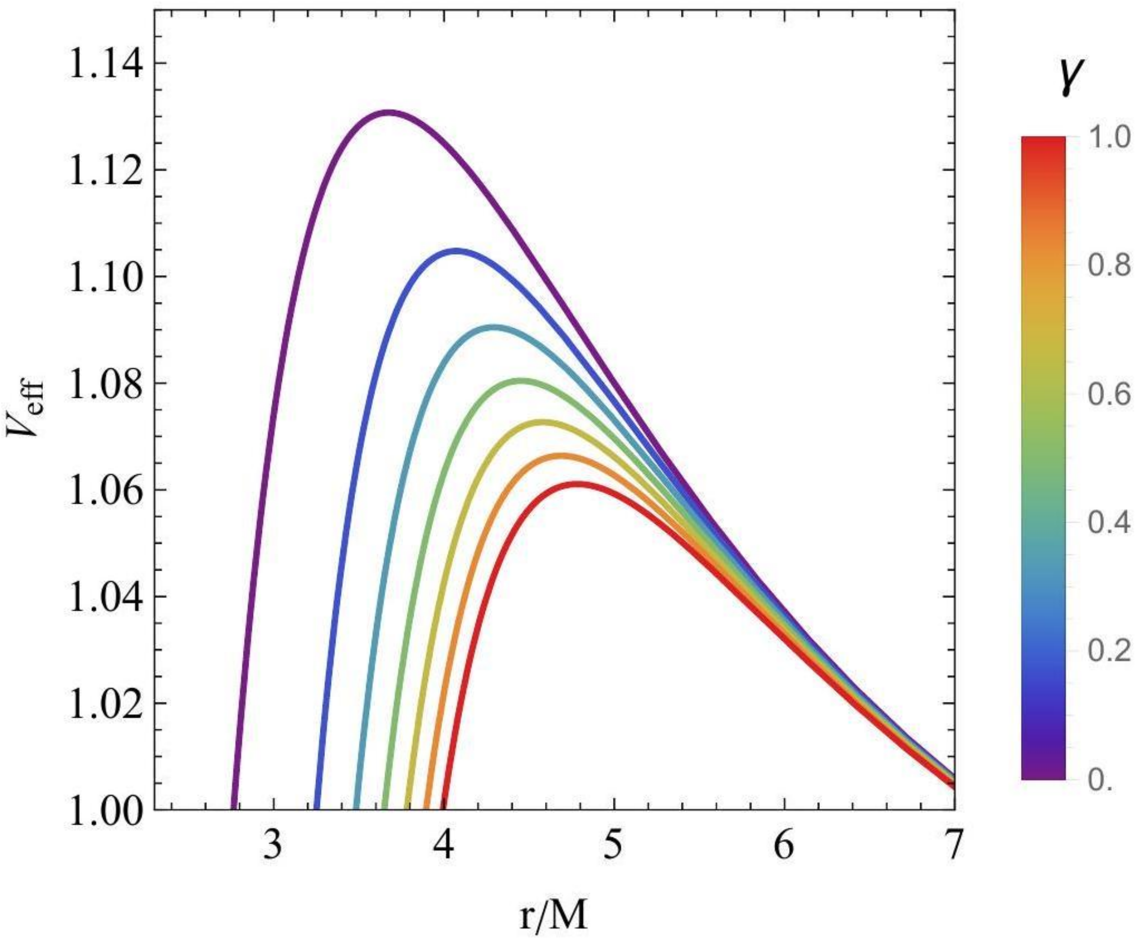

FIG. 2. Effective potential $V_{\text{eff}}(r)$ of a massive test particle for various values of the parameter γ. The behavior of the potential determines the existence of stable and unstable circular orbits.

LIMITING CASES

A. Uncharged Case ($Q=0$)

The horizon equation reduces to

$$r(r^2 - 2M(1-\gamma)) = 0 \tag{15}$$

yielding the physical horizon

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$$r_h=\sqrt{2M(1-\gamma)} \tag{16}$$

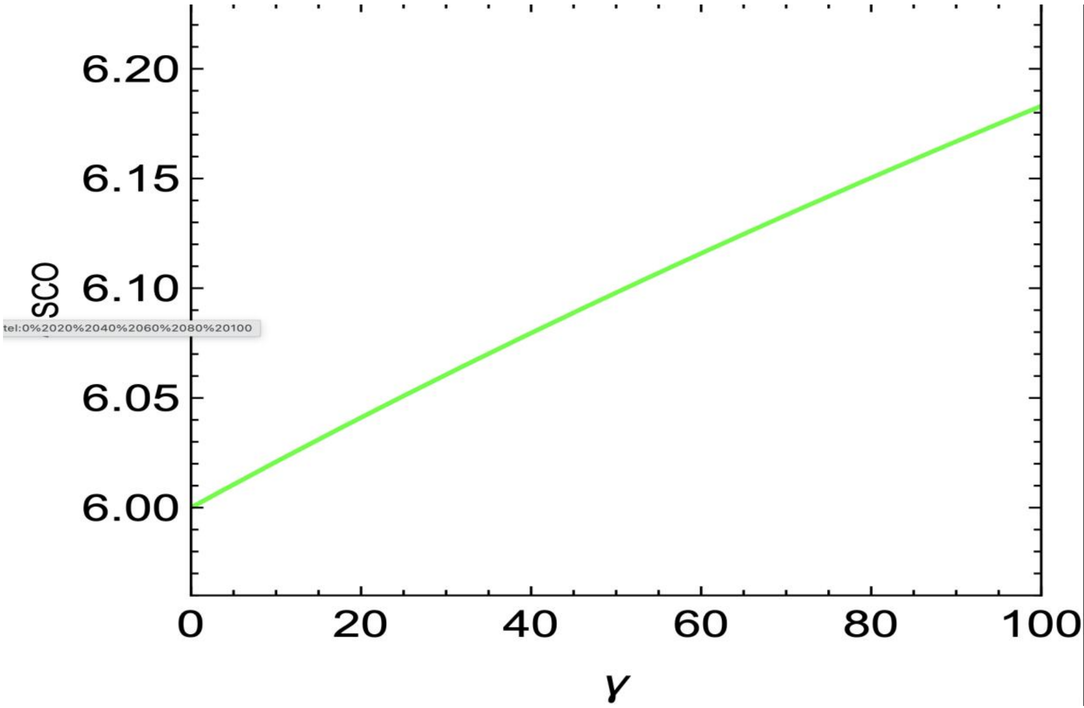

FIG. 3. The plot shows the relationship between the innermost stable circular orbit radius rISCO (vertical axis) and the scalar-tensor coupling parameter γ (horizontal axis). It demonstrates that as γ increases, the value of rISCO also increases, indicating that the scalar field causes stable circular orbits to occur at larger radial distances from the black hole

B. Classical Limit ($\gamma\to 0$)

In the limit $\gamma\to 0$, the metric function becomes

$$F(r)=1-\frac{2M}{r}+\frac{Q^2}{r^2}, \tag{17}$$

corresponding to the Reissner-Nordström black hole.

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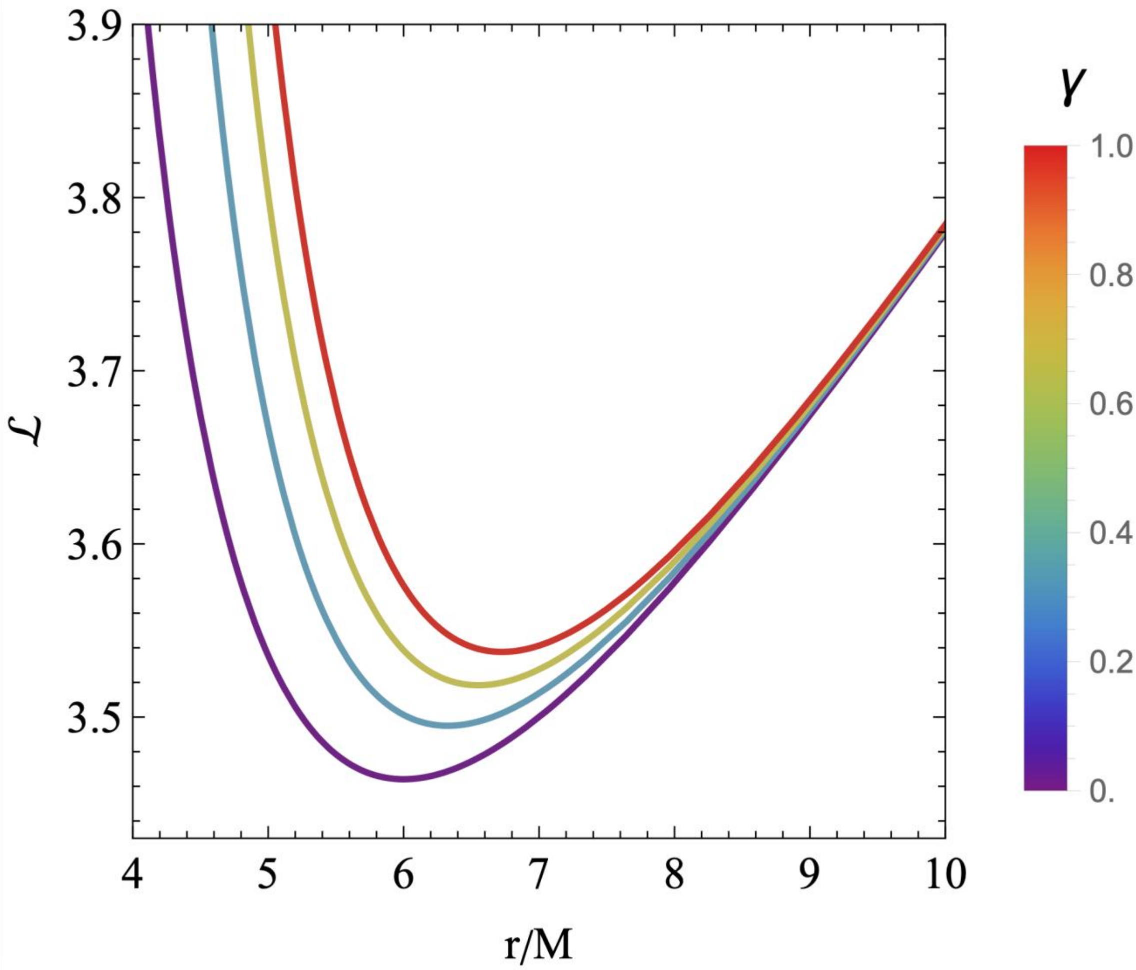

FIG. 4. The plot illustrates the dependence of the angular momentum $\mathcal{L}$ of a test particle on the normalized radial coordinate r/M, for different values of the scalar-tensor coupling parameter γ. The color gradient indicates varying γ from 0 (purple) to 1 (red). As γ increases, the minimum of the angular momentum curve shifts to larger r/M, indicating that the location of the innermost stable circular orbit moves outward.

CONCLUSION

We have derived an exact charged black hole solution characterized by a dimensionless parameter γ. The horizon structure and thermodynamic properties were analyzed in detail. The solution smoothly reduces to the Reissner-Nordström black hole in the appropriate limit, confirming its physical validity stable and unstable circular orbits.

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