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SOLVING OLYMPIAD PROBLEMS

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Abstract

The problems presented in this article help to increase students' interest in solving mathematical olympiad problems, while also developing their logical thinking, creative reasoning, and independent decision-making skills in problem-solving situations.

Keywords

olympiad problems, non-standard problems, creative thinking.

INTRODUCTION

In today's education system, it is important not only to assess students' knowledge level, but also to develop their independent thinking, analytical abilities, and creative approach. This is why olympiad problems are so significant. Studying olympiad problems teaches students not to rely on ready-made formulas to complete test tasks, but to think deeply and solve problems. Olympiad problems are non-standard problems that often cannot be solved directly using traditional methods.

Below we present several olympiad problems with their solutions.

Problem 1. The area of a right triangle S, $r-$ inscribed circle radius, $R-$ circumscribed circle radius $r=\sqrt{S+R^2}-R$

prove that the equality holds.

Solution: If we find the semiperimeter of the right triangle $P=2R+r$ equals.

Then its area is

$$S=\Pr=\left(2R+r\right)r=2rR+r^2$$

$S=2rR+r^2$ Adding to both sides R^2 , we get

$$S+R^2=\left(R+r\right)^2$$

$$\sqrt{S+R^2}=R+r$$

$$r=\sqrt{S+R^2}-R$$

Therefore, the given $r=\sqrt{S+R^2}-R$ equality holds.

has no solution $x=2$ we obtain a unique solution.

Problem 2. Solve the equation.

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$$x^4 + x^2 + \sqrt{2}x + 2 = 0$$

d_1 Solution. In this equation, $x^2 + \sqrt{2}x + 2$ the discriminant of the trinomial is negative.

Therefore, this trinomial is always positive. From this $x^4 + x^2 + \sqrt{2}x + 2 > 0$. Therefore, the equation has no solution.

Problem 3. $y = \sqrt{x} + 4\sqrt{1 - \frac{x}{2}}$. $y_{\max} = ?,\ y_{\min} = ?$.

Solution. $\vec{a}\left(1; 2\sqrt{2}\right),\ \vec{b}\left(\sqrt{x}; \sqrt{2-x}\right)$ we choose vectors. $\vec{a}$ va $\vec{b}$ the angle between vectors α Let. It is evident that, $\vec{a}$ va $\vec{b}$ the coordinates of the vectors are non-negative. We have: $\alpha \quad \left(0; 90^0\right)$ va $0 < \cos\alpha < 1$;

$$\left|\vec{a}\right| = \sqrt{1^2 + \left(2\sqrt{2}\right)^2} = 3;\ \left|\vec{b}\right| = \sqrt{\sqrt{x}^2 + \sqrt{2-x}^2} = \sqrt{2}.$$

From the scalar product of vectors va $0 < \cos\alpha < 1$ using the inequality,
$y = \vec{a}\ \vec{b} = \left|\vec{a}\right|\ \left|\vec{b}\right|\ \cos\alpha \quad \left|\vec{a}\right|\ \left|\vec{b}\right| = 3\sqrt{2}$ va $y = \vec{a}\ \vec{b} = \sqrt{x} + 2\sqrt{2}\sqrt{2-x} \quad 3\sqrt{2}$, if
$\vec{a} = k\vec{b},\ \ k\sqrt{x} = 1,\ k\sqrt{2-x} = 2\sqrt{2}$.

$\vec{b}\left(\sqrt{2}; 0\right)$, $x = 2$ va $y(2) = \sqrt{2}$, the function achieves its minimum value.

Therefore, the maximum value of the given function is $3\sqrt{2}$, the minimum value is $\sqrt{2}$ equals.

Problem 4. Find the integer part of the number.

$$\sqrt[3]{m^3 - m + \sqrt[3]{m^3 - m + \quad + \sqrt[3]{m^3 - m}}} + \sqrt{n^2 - n + \sqrt{n^2 - n + \quad + \sqrt{n^2 - n}}}$$

$m > 0, n > 0, m, n \quad N$.

*Solution.*If the last difference of the first summand is m and the last difference of the second summand is n and we calculate, the sum is $m+n$ less than. On the other hand, the first summand is $\sqrt[3]{m^3 - m}$ greater than, which in turn is $m - \frac{1}{3}$ dan katta, ya'ni $\sqrt[3]{m^3 - m} > m - \frac{1}{3}$.

Bundan $m^3 - m > m^3 - m^2 + \frac{m}{3} - \frac{1}{27}$, ya'ni $27m^2 - 36m + 1 > 0$ the inequality follows.

It is easy to see that $m \quad 2$ da $27m^2 - 36m + 1 > 0$ holds. Similarly, for the second summand $\sqrt{n^2 - n} > n - \frac{2}{3}$ we can see.

Therefore, yig'indi $m+n-1$ dan katta,ya'ni berilgan ifodaning butun qismi

$m+n-1$ equals.

Problem 5. ABCD rectangle, $d_1=d_2$. If $AB=1, BC=\sqrt{2}, CD=\sqrt{3}$ find its angles.

Solution.

Solution. ACC_1A_1 Consider the quadrilateral. Its AA_1 va CC_1 sides are parallel to the diagonal BD of the quadrilateral.

Problem 6. ABC of the right triangle BC The straight line through the midpoint of the leg and the center of the inscribed circle AC intersects the leg M at the point. Prove that this $\frac{AM}{MC}=tg\frac{A}{2}$ equality is true.

Solution.

According to the problem, $CN=NB$ .Let the tangent points of the inscribed circle be E, K va F bo'lsin.

$AC=b, BC=a, AB=c$ Let us denote.

MOF, NOE From the similarity of triangles $\frac{MF}{OF}=\frac{OE}{NE}$ $\quad MF=\frac{OE\ \ OF}{NE}$

Ma'lumki, $AK=p-a, BE=p-b, CF=p-c$ the equalities hold.

From this, $$MF=\frac{r^2}{\frac{a}{2}-r}=\frac{2r^2}{a-2r}=\frac{2(p-c)^2}{c-b}=\frac{c^2-ac+ab-bc}{c-b}$$
$$=c-a=b-2r.$$

Now, $AM=b-(b-2r)-r=r, MC=b-\frac{b+a-c}{2}=p-a$ from the equalities

$\frac{AM}{MC}=\frac{r}{p-a}=tg\frac{A}{2}$ we obtain.

Therefore, $\frac{AM}{MC}=tg\frac{A}{2}$ tenglik holds ekan.

Problem 7. The roots α, β, γ with different coefficients $x^3+ax^2+bx+c=0$ How many equations exist?

Solution. α, β, γ Let be distinct roots of the equation. Then,

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$$\alpha^2 a + \alpha b + c = -\alpha^3,$$
$$\beta^2 a + \beta b + c = -\beta^3,$$
$$\gamma^2 a + \gamma b + c = -\gamma^3$$

we have the equalities.

We calculate all nonzero roots of the equation. The system of equations a, b, c

we find them. Multiply the first equation by β^2 , the second by α^2 and subtract. Then multiply the first by γ^2 , the third by α^2 and subtract. We get:

$$(\alpha^2\beta - \alpha\beta^2)b + (\alpha^2 - \beta^2)c = \alpha^3\beta^2 - \alpha^2\beta^3,$$
$$(\alpha^2\gamma - \alpha\gamma^2)b + (\alpha^2 - \gamma^2)c = \alpha^3\gamma^2 - \alpha^2\gamma^3.$$

$\alpha \quad \beta, \alpha \quad \gamma$ From this system $\begin{matrix} \alpha\beta b + (\alpha+\beta)c = \alpha^2\beta^2, \\ \alpha\gamma b + (\alpha+\gamma)c = \alpha^2\gamma^2 \end{matrix}$ we get.

Finding c from this system, $c = -\alpha\beta\gamma$ As a result of transformations $b = \alpha\beta + \alpha\gamma + \beta\gamma$ va

$a = -(\alpha + \beta + \gamma)$ the equalities follow.

$$x^3 + ax^2 + bx + c = 0 \qquad (1)$$

From equation (1), we have:

$$a + b + c = -a,\ ab + ac + bc = b\ ,\ abc = -c\,. \qquad (2)$$

$c \quad 0$ da $ab = -1$ va (1) , (2) from the equalities

$2a^2 + ac - 1 = 0$, $a^2c - a - c + 1 = 0$ the equalities follow.

$a^2c - a - c + 1 = 0$ tenglamadan $(a^2 - 1)c = a - 1$ and if $a = 1$ bo'lsa,

$b = -1$ va $2a^2 + ac - 1 = 0$ dan $c = -1$ it follows.

This contradicts the condition. Therefore, $a \quad -1$ dan $(a+1)c = 1$, $2a^2 - c = 0$ it follows. $1 = (a+1)c = (a+1)2a^2$

,ya'ni $2a^3 + 2a^2 - 1 = 0$ bo'ladi. (3)

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Ushbu $f(x)=2x^3+2x^2-1$ Taking the derivative, $6x^2+4x=0$ tenglikdan $x=-\frac{2}{3}$

at point $f\ -\frac{2}{3}\ =-\frac{19}{27}$ achieves the maximum.

Therefore, $f(x)=0\ x\ \ R^-$, $.\ [0;\ \ )$ In the interval $f(x)$

$[0;\infty)$ In the interval $f(x)$ the function is monotone, $f(-1)=-1<0$, $f(1)=3>0$. va

Therefore, $f(x)=0$ the equation has one a_0 positive root. a_0 ; $b_0=-\frac{1}{a_0}$ va $c_0=\frac{1}{a_0+1}$

We verify the numbers are distinct. In fact $b_0=a_0$ is incorrect, whereas $c_0=a_0$;

$-a_0=a_0+1;\ a_0=-\frac{1}{2}.$

$-\frac{1}{2}$ soni (3) is not a root of (3). Thus, one unique equation with nonzero roots satisfies the condition. If one root equals zero, then $c=0$, and a va b has a nonzero root $x^2+ax+b=0$ we need to find all equations of the form. By Vieta's theorem $\begin{matrix} a+b=-a, \\ ab=b \end{matrix}$ we get.

$a=1$, $b=-2$ two equations satisfy the condition. $x^3+x^2-2x=0$,

$x^3+a_0x^2-\frac{1}{a_0}x-\frac{1}{a_0+1}=0$

a_0 , $x^3+x^2-2x=0$ is the unique solution.

CONCLUSION

In conclusion, olympiad problems are an integral part of education, as they develop students' independent thinking and serve as an important tool for developing intellectual potential. By studying these problems step by step, high results can be achieved.

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