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IRRATIONAL EQUATIONS INVOLVING SQUARE, CUBE, AND HIGHER-ORDER ROOTS AND METHODS FOR SOLVING THEM

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Abstract: This paper analyzes examples of solving equations that involve roots of the second and higher degrees within the topic *"Irrational Equations."* The study goes beyond the examination of equations containing only square roots and extends to the analysis of equations of higher orders. Furthermore, the purpose of this work is to help secondary school students understand and focus on key aspects that must be considered when solving equations involving irrational expressions.

Keywords: irrational equations, domain of definition, substitution of variables, expression substitution.

INTRODUCTION

We begin this article with the words of René Descartes (1596–1650), the French philosopher and mathematician:

"The discovery of irrational numbers demonstrates the human mind's ability to comprehend even the unseen."

This is because, in this article, we are engaged in demonstrating remarkable results in solving a series of equations that highlight the significance of irrational numbers.

1. $\dfrac{1}{x-\sqrt{x^2-x}}-\dfrac{1}{x+\sqrt{x^2-x}}=\sqrt{3}$.[2,3]

$\dfrac{1}{x-\sqrt{x^2-x}}-\dfrac{1}{x+\sqrt{x^2-x}}=\sqrt{3}$ Here's the English translation of your sentence:

$$\frac{1}{x-\sqrt{x^2-x}}-\frac{1}{x+\sqrt{x^2-x}}=\sqrt{3}$$

$$\frac{x+\sqrt{x^2-x}-(x-\sqrt{x^2-x})}{(x+\sqrt{x^2-x})(x-\sqrt{x^2-x})}=\sqrt{3}$$

$$\frac{x+\sqrt{x^2-x}-(x-\sqrt{x^2-x})}{(x+\sqrt{x^2-x})(x-\sqrt{x^2-x})}=\sqrt{3}$$

$$\frac{x+\sqrt{x^2-x}-x+\sqrt{x^2-x}}{x^2-x^2+x}=\sqrt{3}$$

$$\frac{2\sqrt{x^2-x}}{x}=\sqrt{3}$$

"Now, we solve the final resulting equation."

$$\frac{2\sqrt{x^2-x}}{x}=\sqrt{3} \qquad 2\sqrt{x^2-x}=\sqrt{3}x$$

$$4x^2-4x=3x^2 \qquad x^2-4x=0 \qquad x_1=0,\ x_2=4$$

We obtain the solutions; however, as we noted in [1], we have not determined the domain of the equation. Therefore, we substitute the obtained solutions back into the equation to verify them. It becomes apparent that one of the solutions cannot satisfy the equation. Hence, the equation has a unique solution.

2 $\sqrt{x\sqrt[5]{x}} - \sqrt[5]{x\sqrt{x}} = 56$ [2,4]

$\sqrt{x\sqrt[5]{x}} - \sqrt[5]{x\sqrt{x}} = 56$ To find the solution of the equation, we first combine the irrational expressions under a single root.

$$\sqrt{x\sqrt[5]{x}} - \sqrt[5]{x\sqrt{x}} = 56$$

$$\sqrt{\sqrt[5]{x^5\ x}} - \sqrt[5]{\sqrt{x^2\ x}} = 56$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$$

$$\sqrt{\sqrt[5]{x^5\ x}} - \sqrt[5]{\sqrt{x^2\ x}} = 56$$

$$\sqrt[10]{x^6} - \sqrt[10]{x^3} = 56$$

Now, in this equation, we solve it using the substitution method as in [2,3]. That is $\sqrt[10]{x^3} = t$ we introduce the substitution ...and the equation takes the following form..

$$\sqrt[10]{x^6} - \sqrt[10]{x^3} = 56$$

$$t^2 - t - 56 = 0$$

$$t_1 = -7,\ t_2 = 8$$

$$t_1 = -7$$

$$\sqrt[10]{x^3} = t$$

According to the substitution, a root of even degree cannot be equal to a negative number.

$$\sqrt[10]{x^3} = 8 \qquad x^3 = 8^{10} \qquad x = \sqrt[3]{8^{10}} = (\sqrt[3]{8})^{10} = 2^{10} = 1024$$

$$x = 1024$$

3. $\dfrac{1}{\sqrt{x} + \sqrt[3]{x}} + \dfrac{1}{\sqrt{x} - \sqrt[3]{x}} = \dfrac{1}{3}$ [2,3]

Solution: To solve this equation, we first choose a common denominator and simplify the expression.

$$\frac{1}{\sqrt{x} + \sqrt[3]{x}} + \frac{1}{\sqrt{x} - \sqrt[3]{x}} = \frac{1}{3}$$

$$\frac{2\sqrt{x}}{x - \sqrt[3]{x^2}} = \frac{1}{3}$$

An equality is formed, and by using the property of proportion, we perform further simplifications in the expression.

$$\frac{2\sqrt{x}}{x - \sqrt[3]{x^2}} = \frac{1}{3}$$

$$6\sqrt{x} = x - \sqrt[3]{x^2}$$

$$x - \sqrt[3]{x^2} - 6\sqrt{x} = 0$$

$$\sqrt{x}\left(\sqrt{x} - \sqrt[6]{x} - 6\right) = 0$$

As a result of the simplifications, the expression takes the form shown above. We then set each of the two factors equal to zero to find the solutions of the equation. $\sqrt{x}\left(\sqrt{x} - \sqrt[6]{x} - 6\right) = 0$

$$x_1 = 0,\ \sqrt{x} - \sqrt[6]{x} - 6 = 0$$

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The first solution of the equation is obtained. To find the second solution, we bring the roots inside the parentheses to the same degree and introduce a substitution. $\sqrt[6]{x^3}-\sqrt[6]{x}-6=0 \qquad \sqrt[6]{x}=t$

$t^3-t-6=0 \qquad t^3-8-t+2=0$

$(t-2)(t^2+2t+4-1)=0$

$t_1=2,\ t^2+2t+3=0$

$t^2+2t+3=0$

$x_1=0,\ x_2=64$

$x_1=0$ It cannot be a solution of the equation. Hence, the equation has a unique solution.

4. $\dfrac{2+x}{2-x}+\sqrt{x}=1+x$.[2,3]

Solution: To solve this equation, we first transfer the fractional part of the expression on the left-hand side of the equality to the right-hand side and perform the necessary simplifications.

$$\frac{2+x}{2-x}+\sqrt{x}=1+x$$

$$\sqrt{x}=1+x-\frac{2+x}{2-x}$$

$$\sqrt{x}=1+x+\frac{2+x}{x-2}$$

$$\sqrt{x}=\frac{x^2-x-2+2+x}{x-2}$$

$$\sqrt{x}=\frac{x^2}{x-2}$$

We obtain an equality. Then, in this equality, we use the property of proportion just as in the example above

$$\sqrt{x}=\frac{x^2}{x-2} \qquad x^2=\sqrt{x}(x-2) \qquad \sqrt{x}\left(x\sqrt{x}-x+2\right)=0$$

$\sqrt{x}=t$ By introducing a substitution as above, we transform the equation into the following form.

$$\sqrt{x}\left(x\sqrt{x}-x+2\right)=0 \qquad t(t^3-t^2+2)=0 \qquad t_1=0$$

Then, using the formulas for factoring sums and differences of powers, we split it into parentheses and find its solution

$$t^3-t^2+2=0 \qquad t^3+1-t^2+1=0 \qquad (t+1)(t^2-t+1)-(t+1)(t-1)=0$$

$$(t+1)(t^2-2t+2)=0 \qquad t_2=-1$$

$t_2=-1$ However, we discard this solution because a root of even degree in the substitution cannot be equal to a negative number. From this, it follows that the equation has a unique solution $x=0$

CONCLUSION

For students up to the 10th grade, examples similar to those analyzed and solved in this article have not been presented in mathematics textbooks. Therefore, in this article, we used literature similar to the old M.I. Skanavy *Collection of Problems in Mathematics* to provide students with additional knowledge and to assist them in preparing for various olympiads and competitions. We analyzed several examples that could serve as helpful practice.

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