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SELF-OSCILLATIONS OF A LINEAR VISCOELASTIC ROD FREELY SUPPORTED AT THE ENDS

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Annotation: This article considers the problem of self-oscillations (flutter) of a physically linear viscoelastic rod in a gas flow, taking into account linear dependencies. The statement and solution method of the problem of flutter of a viscoelastic rod simply supported at the ends are given.

Keywords: viscoelasticity, rod, self-oscillations, flutter, physical linearity, aerodynamic linearity, Bubnov-Galerkin method, relaxation kernel, numerical method, critical speed, linear integro-differential equation.

Introduction. The hereditary theory of viscoelasticity has provided a broad opportunity to describe the dynamic processes of deformation of various materials. Since rods are used as structural elements in many branches of industry and technology, the study of their dynamic behavior in various shapes and the study of structures for vibrations and dynamic stability taking into account the physical linearity of the material are relevant.

Despite the presence of numerous works devoted to physically linear vibrations and stability of rods, the flutter of a linear viscoelastic rod has not been sufficiently studied to date.

The Bubnov-Galerkin method for solving stability problems was first proposed in I.G. Bubnov's review of S.P. Timoshenko's work [8]. Subsequently, this method was developed by B.G. Galerkin [9] and is successfully used, including for solving rod stability problems [10].

Statement of the problem. Let us consider the problem of flutter of a viscoelastic rod taking into account physical linearity [4, 9]

$$\sigma = m_1(1 - R^*)\varepsilon, \quad \varepsilon = u_x, \; u =- zw_x \qquad (1)$$

or

$$\sigma =- m_1(1 - R^*)zw_x \qquad (2)$$

where m_1 - is the elastic constant.

Taking into account also the influence of aerodynamic linearity, according to the one-dimensional theory of gas, the gas pressure on the piston had (4). Applying the Newton binomial formula to equation (4) and in the first approximation we obtain [1]:

$$q = \frac{\chi p}{c} \quad V\frac{\partial w}{\partial x} + \frac{\partial w}{\partial t}$$

where indicated $q = p - p$, $k = \frac{\chi p}{c}$

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$$q = k\ V\frac{\partial w}{\partial x} + \frac{\partial w}{\partial t} \qquad (3)$$

Let us solve the flutter problem in a linear viscoelastic formulation taking into account physical and aerodynamic linearities. For this purpose, we will construct a mathematical model for studying a viscoelastic rod in a gas flow taking into account these linearities.

In this case, accepting the hypothesis of flat sections for the bending moment, we use the following formula [2]:

$$M_x = \int_{-h/2}^{h/2} b(x)\sigma_x z dz \qquad (4)$$

(2) put into (4) and get

$$M_x = -b(x)\left(1 - R^*\right)\int_{-h/2}^{h/2} m_1 z w_{xx} z dz = -b(x)\left(1 - R^*\right)m_1 w_{xx}\int_{-h\backslash 2}^{h/2} z^2 dz =$$

$$= -b(x)\left(1 - R^*\right)m_1 w_{xx}\frac{z^3}{3}\Bigg|_{-h\backslash 2}^{h\backslash 2} = -b(x)\left(1 - R^*\right)m_1 w_{xx}\frac{h^3}{12} =$$

$$= -\left(1 - R^*\right)m_1\frac{b(x)h^3(x)}{12}w_{xx}$$

$$M_x = -E(1 - R^*)J_2 w_{xx} \qquad (5)$$

and equal for rods of width b(x) and height h(x)

$$J_2 = \frac{b(x)h^3(x)}{12}.$$

Substituting (5) into the equilibrium equation [2] and moving to dimensionless coordinates and omitting the primes, we have

$$-\left(1-R^*\right)\frac{\partial}{\partial x^2}\ \frac{m_1 J_2^0 h_0}{a^2}g(x)w_{xx}\ = m_0 F(x)\frac{h_0}{t_1^2}w_{tt} + kV\frac{h_0}{a}w_x + kz\frac{h_0}{t_1}w_t$$

$$-m_1 J_2^0\frac{h_0}{a^4}\left(1-R^*\right)\frac{\partial}{\partial x^2}\left[g(x)w_{xx}\right] = m_0 F(x)\frac{h_0}{t_1^2}w_{tt} + kV\frac{h_0}{a}w_x + kz\frac{h_0}{t_1}w_t$$

$$\left(1-R^*\right)\frac{\partial}{\partial x^2}\left[g(x)w_{xx}\right] + F(x)w_{tt} + Pw_x + \gamma w_t = 0 \qquad (6)$$

where

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$$w = h_0\overline{w},\ x = a\overline{x},\ t = t_1\overline{t},\ m(x) = m_0\overline{F(x)},\ h(x) = h_0\overline{h(x)},\ b(x) = b_0\overline{b(x)},$$

$$J_2 = J_2^0 g(x), \quad g(x) = b(x)h^3(x), \quad J_2^0 = \frac{b_0 h_0^3}{12},$$

$$P = \frac{kVa^3}{m_1 J_2^{(0)}}, \quad t_1 = \sqrt{(m_0 a^4)/(m_1 J_2^{(0)})}, \quad \gamma = \frac{kza^4}{m_1 J_2^{(0)} t_1}, F(x) = b(x)h(x)$$

$b(x)=c-a_1x;\ h(x)=1-a_2x;\ c=5$

h_0 - is the value of the rod height at the ends, b_0 - is the value of the rod width at the ends, m_0 - is the value of the mass corresponding to a unit variable cross-section of the rod.

Linear partial differential equations (6), together with the boundary [5] and initial conditions, represent a mathematical model of the problem of flutter of a linear viscoelastic rod. It is required to find the critical speeds P_{cr} leading to an increasing amplitude of oscillations.

We will construct an approximate solution using the Bubnov-Galerkin method. We will represent the solution of the partial differential equation (6) in the form

$$w = \sum_{k=1}^{N} u_k(t)\varphi_k(x) \qquad (7)$$

where $\varphi_\kappa(x)$ - are known, basis functions satisfying the given boundary conditions, $u_\kappa(t)$ - are unknown functions of time to be determined.

To find the unknown functions $u_\kappa(t)$, we substitute (7) into (6)

$$w_x = \sum_{k=1}^{N} u_k(t)\varphi_k(x); \quad w_{xx} = \sum_{k=1}^{N} u_k(t)\varphi_k(x);$$

$$w_t = \sum_{k=1}^{N} \dot{u}_k(t)\varphi_k(x); \quad w_{tt} = \sum_{k=1}^{N} \ddot{u}_k(t)\varphi_k(x).$$

$$(1-R^*)\frac{\partial}{\partial x^2} g(x)\sum_{k=1}^{N} u_k(t)\varphi_k(x) + F(x)\sum_{k=1}^{N} \ddot{u}_k(t)\varphi_k(x) + P\sum_{k=1}^{N} u_k(t)\varphi_k(x) +$$
$$+\gamma\sum_{k=1}^{N} \dot{u}_k(t)\varphi_k(x) = 0$$

$$(1-R^*)\frac{\partial}{\partial x^2} g(x)\sum_{k=1}^{N} u_k(t)\varphi_k(x) + F(x)\sum_{k=1}^{N} \ddot{u}_k(t)\varphi_k(x) + P\sum_{k=1}^{N} u_k(t)\varphi_k(x) +$$
$$+\gamma\sum_{k=1}^{N} \dot{u}_k(t)\varphi_k(x) = 0$$

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$$(1-R^*)\sum_{k=1}^{N} u_k(t)[g(x)\varphi_k(x)] + F(x)\sum_{k=1}^{N}\ddot{u}_k(t)\varphi_k(x) + P\sum_{k=1}^{N}u_k(t)\varphi_k(x) + \gamma\sum_{k=1}^{N}\dot{u}_k(t)\varphi_k(x) = 0$$

multiplying by $\varphi_i(x)$ and integrating over x

$$(1-R^*)\sum_{k=1}^{N} u_k(t)\int_0^1 [g(x)\varphi_k(x)]\,\varphi_i(x)dx + \sum_{k=1}^{N}\ddot{u}_k(t)\int_0^1 F(x)\varphi_k(x)\varphi_i(x)dx +$$

$$+P\sum_{k=1}^{N}u_k(t)\int_0^1 \varphi_k(x)\varphi_i(x)dx + \gamma\sum_{k=1}^{N}\dot{u}_k(t)\int_0^1 \varphi_k(x)\varphi_i(x)dx = 0$$

For integrals we introduce notations and obtain the following linear systems of ordinary integro-differential equations

$$\sum_{k=1}^{N}\left[a_{ki}\ddot{u}_k(t) + \gamma b_{ki}\dot{u}_k(t) + \omega_{ki}(1-R^*)u_k(t) + Pd_{ki}u_k(t)\right] = 0,\ i = \overline{1,N} \qquad (8)$$

where

$$a_{ki} = \int_0^1 F(x)\varphi_k(x)\varphi_i(x)dx, \qquad b_{ki} = \int_0^1 \varphi_k(x)\varphi_i(x)dx,$$

$$\omega_{ki} = \int_0^1 d(x)\varphi_k(x)\,\varphi_i(x)dx, \qquad d_{ki} = \int_0^1 \varphi_k(x)\varphi_i(x)dx,$$

Integration of the linear system (8) with the Rzhanitsyn–Koltunov kernel $R(t)=A\cdot e^{-\beta t}t^{\alpha-1}$, $A>0$, $\beta>0$, $0<\alpha<1$ in wide ranges of change of physical and mechanical parameters of the rod, was performed by a numerical method based on analytical transformations [3]. According to this method, the numerical values of the sought functions $u_k(t_l)=u_{k,l}$ are found from the solution of the following recurrent system of linear algebraic equations

$$\sum_{k=1}^{N}\left(a_{ki} + \gamma\frac{\Delta t}{2}b_{ki}\right)u_{k,l} = \sum_{k=1}^{N}\left[(a_{ki} + \gamma t_l b_{ki})u_{k,0} + t_l a_{ki}\dot{u}_{k,o}\right] - \sum_{k=1}^{N}\sum_{i_1=1}^{l-1}\Big[\gamma A_{i_1}b_{ki}u_{k,i_1} + A_{i_1}(t_l - t_{i_1})$$

$$\Big(\omega_{ki}\Big(u_{k,i_1} - \frac{A}{2}\sum_{i_2=1}^{i_1}B_{i_2}e^{-\beta t_{i2}}u_{k,i_1-i_2+1}\Big) + Pd_{ki}u_{k,i_1}\Big)\Big],\ i = \overline{1,N} \qquad (9)$$

where

$$t_i = i\Delta t,\ B_1 = \frac{\Delta t^\alpha}{2},\ B_{i_2} = \frac{\Delta t^\alpha\left[(i_2+1)^\alpha - (i_2-1)^\alpha\right]}{2},\ i_2 = \overline{2, i_1 - 1}$$

$$B_{i_1} = \frac{\Delta t^\alpha\left[i_1^\alpha - (i_1-1)^\alpha\right]}{2},\ A_1 = \frac{\Delta t}{2},\ A_i = \Delta t,\ i_1 = \overline{2, i-1},\ i = 1,2.....$$

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The calculation was carried out for various rheological parameters and rod shapes in the plan. The calculation was made for both an ideally elastic and a viscoelastic rod.

Beam functions are taken as the basis functions φк(x) of a rod simply supported at the ends

$$\varphi_k(x) = \sin\lambda_k x; \quad \lambda_k = k\pi$$

and for the initial conditions

$$u_k(0) = \int_0^1 \alpha_0(x)\varphi_k(x)dx, \quad \dot{u}_k(0) = 0 \qquad \text{где } \alpha_0(x) = \{[x(1-x)]^4 + \varphi_1(x)\}/100$$

Analysis and conclusion. Analysis of the results of physically nonlinear problems, given in the table, shows that the critical speed is determined by the linear theory both in ideally elastic and in viscoelastic formulations, and is only the upper limit of critical speeds for real structures.

The cross-section of the beam changes according to the law $b(x) = c - \alpha_1 x$; $h(x) = 1 - \alpha_2 x$; where c=5. N - is the number of terms in the solution.

N	A	α	β	α_1	α_2	γ	P_{cr}
2 3 4	0.0	0.25	0.05	4.0	0.2	0	646,577 717,590 743,270
2 3 4	0.05	0.25	0.05	4.0	0.2	0	297,34 282,56 311,47
2	0.01 0.03 0.07 0.1	0.25	0.05	4.0	0.2	0	378,40 317,54 275,73 266,15
2	0.05	0.15 0,35 0.5	0.05	4.0	0.2	0	265,07 303,44 326,18
2	0.05	0.25	0.05	4.0	0.2	0,5 1,0 2,0	352,81 402,23 463,61

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When studying the influence of parameters, it is sufficient to consider the number of terms in the solution N = 2. A decrease in the critical velocity of the elastic state (Pcr = 646.57) is observed relative to the viscous state (Pcr = 297.34), which shows 45.9%. The influence of parameters **A, α** and **γ** (see table) were studied. The effects caused by taking into account the hereditary properties of the rod material on the critical velocity, in the linear formulation turned out to be significant, for example, an insignificant decrease or increase in the singularity parameter **α** leads to a significant decrease or increase, an insignificant increase in the viscosity parameter **A** leads to a significant decrease, an insignificant increase in the parameter **γ** leads to a significant increase in the critical flutter velocity. Consequently, taking into account viscoelasticity is of great importance, since the variation in the parameters of the viscous properties of the material shows the level of intensity of dissipative processes in these structures.

Conclusion.

The main results of the work are as follows:

1. Based on the cubic theory of viscoelasticity, a boundary value problem for the dynamic calculation of a rod made of a physically nonlinear viscoelastic material was formulated.

2. A general computational algorithm for constructing the initial relationships of the Bubnov-Galerkin method as applied to boundary value problems for the dynamic calculation of a rod was developed and implemented on a computer.

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