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DIAMETRAL AND SYMMETRY PLANE OF A SURFACE

Annotation: This article discusses the concepts of diametral and symmetry planes of surfaces, their role and significance in geometry, as well as their practical applications. A diametral plane of a surface is a plane that intersects the surface and divides it into two symmetric parts. Symmetry planes, on the other hand, divide the surface into two parts, each of which is a mirror image of the other. These concepts play an important role in analyzing the symmetry of geometric objects, their construction, and optimization. The article also covers the use of these planes in various geometric shapes and models, their significance in engineering problems based on symmetry principles, and the mathematical methods used to determine the properties of surfaces.

Keywords: Surface of diametral and symmetrical planes, parallel chords, differentiation, vector, system of equations.

INTRODUCTION

We have introduced the concept of diametral lines for second-order curves. For a second-order surface, we introduce the concept of a diametral plane. If a straight line intersects a second-order surface at two points, M and N, then the segment MN is called a chord of the second-order surface.

Theorem 1. The midpoints of parallel chords lie in the same plane.[16-19]

Definition 1. A plane passing through the midpoints of parallel chords is called the diametral plane of the surface.[10-15]

To write the equation of the diametral plane in an arbitrary Cartesian coordinate system, we use the following expressions:

$$x = \bar{x} + \lambda t,\ y = \bar{y} + \mu t,\ z = \bar{z} + \gamma t \text{ va } x = \bar{x} - \lambda t,\ y = \bar{y} - \mu t,\ z = \bar{z} - \gamma t \quad (1)$$

$$F(x, y, z) = 0$$

we obtain the equation:

$$2F(x, y, z) \pm 2t\big(\lambda F_x(x, y, z) + \mu F_y(x, y, z) + \gamma F_z(x, y, z)\big) +$$
$$+ t^2(a_{11}\lambda^2 + a_{22}\mu^2 + a_{33}\lambda^2 + 2a_{12}\lambda\mu + 2a_{23}\mu\gamma + 2a_{31}\lambda\gamma) = 0 \quad (2)$$

For this equation to hold for any t, the coefficient of t must be zero. Therefore, the equation of the diametral plane can be written as:

$$\lambda F_x + \mu F_y + \gamma F_z = 0 \quad (3)$$

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It is evident that if a second-order surface has a center of symmetry, then any diametral plane passes through this center.[1-9]

Thus, the center of a second-order surface is determined by solving the system of equations:

$F_x = 0, \ \ F_y = 0, \ F_z = 0$ (4)

For a paraboloid, the diametral plane is parallel to its axis. In this case, since $a_{33} = 0$, equation (3) does not contain the variable z.

For elliptic and hyperbolic cylinders, all points on their axes serve as centers, so any diametral plane passes through the surface.

Example 1: Find the equation of the diametral plane of the second-order surface given by the equation $69x^2 + 3y^2 - 4z^2 + 5xy - 2xz + 4y = 0$ that passes through the line: $\frac{x-2}{3} = \frac{y}{2} = \frac{z+1}{4}$.

Solution:

Given the equation:

$$\lambda F_x + \mu F_y + \gamma F_z = 0$$

For the surface equation:

$$x^2 + 3y^2 - 4z^2 + 5xy - 2xz + 4y = 0$$

we compute the partial derivatives:

$F_x = 2x + 5y - 2z, \ F_y = 6y + 5x + 4, F_z =- 8z - 2x$

Substituting these into the equation:

$$\lambda(2x + 5y - 2z), + \mu(6y + 5x + 4) - \gamma(8z + 2x\,) = 0$$

$$(2\lambda + 5\mu - 2\gamma)x + (5\lambda + 6\mu)y - (2\lambda + 8\gamma)z + 4\mu = 0$$

The normal vector of the plane is:

$$\bar{n} = (2\lambda + 5\mu - 2\gamma, 5\lambda + 6\mu, 2\lambda + 8\gamma)$$

$\frac{x-2}{3} = \frac{y}{2} = \frac{z+1}{4}$

Given the direction vector of the line:

$\bar{u} = (3; 2; 4) \quad \bar{n} \perp \bar{u}$

Since $\bar{n} \perp \bar{u}$, we obtain the equation:

$$6\lambda + 15\mu - 6\gamma + 10\lambda + 12\mu + 8\lambda + 32\gamma = 0$$

$24\lambda + 27\mu + 26\gamma = 0$

The plane passes through the point$M(2; 0; -1)$, thus:

$$2 \cdot (2\lambda + 5\mu - 2\gamma) + 0 \cdot (5\lambda + 6\mu) + (2\lambda + 8\gamma) + 4\mu = 0$$

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$6\lambda + 14\mu + 4\gamma = 0 \quad 3\lambda + 7\mu + 2\gamma = 0$

Rewriting the system of equations:

$$\begin{cases} 24\lambda + 27\mu + 26\gamma = 0 \\ 3\lambda + 7\mu + 2\gamma = 0 \end{cases}$$

Solving for μ and λ:

$29\mu - 10\gamma = 0 \qquad \mu = \frac{10\gamma}{29},$

$3\lambda + \frac{70\gamma}{29} + 2\gamma = 0 \quad 3\lambda = -\frac{128\gamma}{29} \quad \lambda = -\frac{128\gamma}{87},$

Substituting into the equation:

$\left(-\frac{256\gamma}{87} + \frac{50\gamma}{29} - 2\gamma\right)x + \left(-\frac{640\gamma}{87} + \frac{60\gamma}{29}\right)y - \left(-\frac{256\gamma}{87} + 8\gamma\right)z + \frac{40\gamma}{29} = 0$

$\left(-\frac{280\gamma}{87}\right)x + \left(-\frac{460\gamma}{87}\right)y - \left(\frac{440\gamma}{87}\right)z + \frac{40\gamma}{29} = 0$

$14x + 23y + 22z - 6 = 0$

Answer: $14x + 23y + 22z - 6 = 0$

Definition 2:

Let the plane α be given. If for any point M belonging to the surface, the point symmetric to M with respect to the plane α also belongs to the surface, then α is called the symmetry plane of the surface.

Using the equation of the diametral plane, we derive the equation of the symmetry plane. Since the midpoints of mutually parallel chords perpendicular to the symmetry plane belong to the symmetry plane, the equation of the symmetry plane perpendicular to the vector $\bar{a} = \{l, m, n\}$ is given by

$$lF_x + mF_y + nF_z = 0 \qquad (5)$$

Since the vector $\bar{a} = \{l, m, n\}$ is perpendicular to the symmetry plane, the proportionality

$$\frac{a_{11}l + a_{12}m + a_{13}n}{l} = \frac{a_{21}l + a_{22}m + a_{23}n}{m} = \frac{a_{31}l + a_{32}m + a_{33}n}{n} \qquad (6)$$

To determine the direction perpendicular to the symmetry plane from Equation (5), we introduce a proportionality factor k in Equation (6) and obtain the equivalent system:

$$\begin{cases} (a_{11} - k)l + a_{12}m + a_{13}n = 0 \\ a_{21}l + (a_{22} - k)m + a_{23}n = 0 \\ a_{31}l + a_{32}m + (a_{33} - k)n = 0 \end{cases} \qquad (7)$$

Since l, m, n are not all simultaneously zero, the determinant equation

$$\begin{vmatrix} a_{11} - k & a_{12} & a_{13} \\ a_{21} & a_{22} - k & a_{23} \\ a_{31} & a_{32} & a_{33} - k \end{vmatrix} = 0$$

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must hold.

Solving for k from this equation and substituting it into system (7), we determine the direction $\{l, m, n\}$.

If the symmetry plane of the surface is known, it becomes convenient to simplify the second-order surface equation, making it easier to determine the canonical coordinate system.

Definition 2:

Find the symmetric equations of the plane for the given equation:

$$36x^2 + 4y^2 + 40z^2 + 8yz + 36x - 72z + 9 = 0$$

Give coefficients:

$$a_{11} = 36, a_{12} = 0, a_{13} = 0, a_{21} = 0, a_{22} = 4, a_{23} = 4, a_{31} = 0, a_{32} = 4,$$
$$a_{33} = 40$$

The determinant equation:

$$\begin{vmatrix} 36-k & 0 & 0 \\ 0 & 4-k & 4 \\ 0 & 4 & 40-k \end{vmatrix} = 0$$

Expanding:

$$(36-k)(4-k)(40-k) - 16(36-k) = 0$$

$$(36-k)(160 - 44k + k^2 - 16) = 0 \qquad (36-k)(144 - 44k + k^2) = 0$$

Solving for k:

$$k_1 = 36, \qquad k^2 - 44k + 144 = 0$$

$$D = \sqrt{44^2 - 4 \cdot 144} = \sqrt{1360} = 4\sqrt{85}$$

$$k_2 = \frac{44+4\sqrt{85}}{2} = 22 + 2\sqrt{85}, \qquad k_3 = \frac{44-4\sqrt{85}}{2} = 22 - 2\sqrt{85}$$

Thus, we obtain three symmetry planes.

First symmetry plane (for $k_1 = 36$):

Solving:

$$\begin{cases} -32m + 4n = 0 \\ 4m + 4n = 0 \end{cases}$$

$$\begin{cases} 8m - n = 0 \\ m + n = 0 \end{cases} \quad m = 0, n = 0 \quad l \in \mathbb{R}$$

$$lF_x + mF_y + nF_z = 0 \qquad lF_x = 0$$

$$F_x = 0 \qquad F_x = 72x + 36 = 0 \qquad 2x + 1 = 0$$

Thus, the first symmetry plane is:

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$$2x + 1 = 0$$

Second symmetry plane (for $k_2 = 22 + 2\sqrt{85}$):

Solving:

$$\begin{cases} \left(14 - 2\sqrt{85}\right)l = 0 \\ \left(-18 - 2\sqrt{85}\right)m + 4n = 0\,, \quad l = 0 \\ 4m + \left(18 - 2\sqrt{85}\right)n = 0 \end{cases}$$

$$\begin{cases} \left(9 + \sqrt{85}\right)m - 2n = 0 \\ 2m + \left(9 - \sqrt{85}\right)n = 0 \end{cases} \quad m, n \neq 0$$

$$n = 1, \quad \left(9 + \sqrt{85}\right)m = 2 \quad m = \frac{2}{(9+\sqrt{85})} \quad m = \frac{\sqrt{85}-9}{2}$$

Substituting:

$$lF_x + mF_y + nF_z = 0 \qquad l = 0, m = \frac{\sqrt{85}-9}{2}\,,\ n = 1$$

$$\frac{\sqrt{85}-9}{2}F_y + F_z = 0 \qquad F_y = 8y + 8z \qquad F_z = 80z + 8y - 72$$

$$\frac{\sqrt{85}-9}{2}(8y + 8z) + (80z + 8y - 72) = 0$$

$$\left(\sqrt{85} - 9\right)(y + z) + 2(10z + y - 9) = 0$$

$$\sqrt{85}y + \sqrt{85}z - 9y - 9z + 20z + 2y - 18 = 0$$

$$\left(\sqrt{85} - 7\right)y + \left(\sqrt{85} + 11\right)z - 18 = 0\,.$$

Thus, the second symmetry plane is:

$$\left(\sqrt{85} - 7\right)y + \left(\sqrt{85} + 11\right)z - 18 = 0$$

Third symmetry plane (for $k_3 = 22 - 2\sqrt{85}$)

Solving:

$$\begin{cases} \left(14 + 2\sqrt{85}\right)l = 0 \\ \left(-18 + 2\sqrt{85}\right)m + 4n = 0\,, \quad l = 0 \\ 4m + \left(18 + 2\sqrt{85}\right)n = 0 \end{cases}$$

$$\begin{cases} \left(9 - \sqrt{85}\right)m - 2n = 0 \\ 2m + \left(9 + \sqrt{85}\right)n = 0 \end{cases} \quad m, n \neq 0$$

$$n = 1, \quad \left(9 - \sqrt{85}\right)m = 2 \quad m = \frac{2}{(9-\sqrt{85})} \quad m = -\frac{\sqrt{85}+9}{2}$$

Substituting:

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$$\mathrm{l}F_x + mF_y + nF_z = 0 \qquad l = 0,\ m = -\frac{\sqrt{85}+9}{2},\ n = 1$$

$$-\frac{\sqrt{85}+9}{2}F_y + F_z = 0 \qquad F_y = 8y + 8z \qquad F_z = 80z + 8y - 72$$

$$-\frac{\sqrt{85}+9}{2}(8y + 8z) + (80z + 8y - 72) = 0$$

$$-\left(\sqrt{85} + 9\right)(y + z) + 2(10z + y - 9) = 0$$

$$-\sqrt{85}y - \sqrt{85}z - 9y - 9z + 20z + 2y - 18 = 0$$

$$\left(\sqrt{85} + 7\right)y + \left(\sqrt{85} - 11\right)z + 18 = 0$$

Thus, the third symmetry plane is:

$$\left(\sqrt{85} + 7\right)y + \left(\sqrt{85} - 11\right)z + 18 = 0$$

Answer: $2x + 1 = 0,\ \left(\sqrt{85} - 7\right)y + \left(\sqrt{85} + 11\right)z - 18 = 0,\ \left(\sqrt{85} + 7\right)y + \left(\sqrt{85} - 11\right)z + 18 = 0$

Conclusion and Suggestions

In this article, the concepts of diametral and symmetry planes of a surface, their geometric properties, and practical applications were discussed. The symmetry plane serves as an important tool in studying the structure of a surface and analyzing its properties. The diametral plane, on the other hand, is widely used in determining spatial relationships on the surface and in modeling processes. The results of this study can be useful in the analysis of various geometric shapes, as well as in solving practical problems in engineering, technology, and physics. Based on this, an in-depth analysis of symmetry and diametral planes of surfaces may lead to new approaches in scientific and technological fields.

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