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PROBLEM FOR THE THREE-DIMENSIONAL HEAT EQUATION IN AN UNBOUNDED AND FINITE CYLINDER

Annotation: This paper investigates the solution of the problem for three-dimensional heat equation in an unbounded and finite cylinder.

Key words: heat equation , temperature , coordinate, radius, boundary conditions, initial conditions, Fourier method, coefficients, Bessel function .

Let us consider a more general case when the initial temperature is a function of all three coordinates r, θ and z.

If we limit ourselves to the study of the case when the temperature on the lateral surface of the cylinder is maintained equal to zero, then the problem is reduced to solving the equation

$$\frac{\partial u}{\partial t}=a^2\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}+\frac{\partial^2 u}{\partial z^2}\right) \qquad (1)$$

under the boundary condition

$$u\big|_{r=R}=0 \qquad (2)$$

and under the initial condition

$$u\big|_{t=0}=\varphi(r,\theta,z). \qquad (3)$$

We will look for particular solutions of equation (2.3.1) in the form of a product

$$u(r,\theta,z,t)=T(t)\ \omega(r)\ \Phi(\theta)\ Z(z). \qquad (4)$$

Substituting (4) into equation (1), we obtain

$$\frac{T'(t)}{a^2T(t)}=\frac{\omega''(r)+\frac{1}{r}\omega'(r)}{\omega(r)}+\frac{1}{r^2}\frac{\Phi''(\theta)}{\Phi(\theta)}+\frac{Z''(z)}{Z(z)},$$

where

$$Z''(z)+\lambda^2Z(z)=0, \qquad \Phi''(\theta)+m^2\Phi(\theta)=0,$$

$$\omega''(r)+\frac{1}{r}\omega'(r)+\left(k^2-\frac{m^2}{r^2}\right)\omega(r)=0,$$

$$T'(t)+a^2(k^2+\lambda^2)T(t)=0,$$

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where λ, m and k – arbitrary constants.

The general solutions of these equations are as follows:

$$Z(z) = C_1 \cos\lambda z + C_2 \sin\lambda z,$$

$$\Phi(\theta) = C_3 \cos m\,\theta \; + C_4 \sin m\,\theta, \qquad (5)$$

$$\omega(r) = C_5 J_m(kr) + C_6 Y_m(kr),$$

$$T(t) = C_7 e^{-a^2(k^2+\lambda^2)t}.$$

Since the temperature inside the cylinder is obviously a periodic function of the angle θ c 2π, period, then the constant m must be an integer. Further, the constant C_6 must be equal to zero, since otherwise the temperature will go to infinity on the axis of the cylinder, which is, of course, impossible. As for the constant, k, it is determined from the boundary condition (2), which leads to the equation $J_m(kR) = 0.$ It follows that the constant k has an infinite set of values, determined by the formula

$$k_{mi} = \frac{\mu_{mi}}{R} \qquad (i = 1,\ 2,\ 3,\ \dots\ ), \qquad (6)$$

Where $\mu_{m1},\ \mu_{m2},\ \mu_{m3},\ \dots$ – positive roots of the equation $J_m(\mu) = 0.$

Since there are no restrictions on the parameter λ , it can be considered completely arbitrary, changing continuously in the interval from – to + .

From these formulas follows the expressions of the desired functions $A_{0i}(\lambda)$, $B_{0i}(\lambda)$, $A_{mi}(\lambda)$, $B_{mi}(\lambda)$, $C_{mi}(\lambda)$, $D_{mi}(\lambda)$, namely

$$A_{mi}(\lambda) = \frac{1}{\delta_m \pi^2 R^2 J_{m+1}^2(\mu_{mi})} \int_0^R \int_0^{2\pi} \int_{-}^{+} r\varphi(r,\theta_1,\xi) \times$$

$$\times J_m\left(\frac{\mu_{mi} r}{R}\right) \cos m\theta_1 \cos\lambda\xi \, dr d\theta_1 d\xi,$$

$$B_{mi}(\lambda) = \frac{1}{\delta_m \pi^2 R^2 J_{m+1}^2(\mu_{mi})} \int_0^R \int_0^{2\pi} \int_{-}^{+} r\varphi(r,\theta_1,\xi) \times$$

$$\times J_m\left(\frac{\mu_{mi} r}{R}\right) \cos m\theta_1 \cos\lambda\xi \, dr d\theta_1 d\xi,$$

eISSN: 2349-5715 pISSN: 2349-5707

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$$C_{mi}(\lambda)=\frac{1}{\delta_m\pi^2R^2J_{m+1}^2(\mu_{mi})}\int_0^R\int_0^{2\pi}\int_{-\infty}^{+\infty} r\varphi(r,\theta_1,\xi)\times$$

$$\times J_m\left(\frac{\mu_{mi}r}{R}\right)\cos m\theta_1\cos\lambda\xi\, dr d\theta_1 d\xi,$$

$$D_{mi}(\lambda)=\frac{1}{\delta_m\pi^2R^2J_{m+1}^2(\mu_{mi})}\int_0^R\int_0^{2\pi}\int_{-\infty}^{+\infty} r\varphi(r,\theta_1,\xi)\times$$

$$\times J_m\left(\frac{\mu_{mi}r}{R}\right)\cos m\theta_1\cos\lambda\xi\, dr d\theta_1 d\xi,$$

where $\delta_0=2$ and $\delta_m=1$ at $m>0$.

Using the integral

$$\int_{-\infty}^{+\infty} e^{-\alpha^2\lambda^2}\cos\beta\lambda\, d\lambda=\frac{\sqrt{\pi}}{\alpha}e^{-\frac{\beta^2}{4\alpha^2}},$$

it is easy to verify that the solution to problems (1) – (3) can be rewritten in the form

$$u=\frac{1}{a\pi R^2\sqrt{\pi t}}\sum_{m=0}^{\infty}\sum_{i=1}^{\infty} e^{-\left(\frac{a\mu_{mi}}{R}\right)^2 t}\frac{J_m\left(\frac{\mu_{mi}r}{R}\right)}{\delta_m J_{m+1}^2(\mu_{mi})}\times$$

$$\times\int_0^R\int_0^{2\pi}\int_{-\infty}^{+\infty}\rho\varphi(\rho,\theta_1,\xi)e^{-\frac{(z-\xi)^2}{4a^2t}}J_m\left(\frac{\mu_{mi}\rho}{R}\right)\cos m(\theta-\theta_1)d\rho d\theta_1 d\xi. \quad (7)$$

Let us now consider the problem of heat propagation in a circular cylinder of radius R and height $2h$, whose initial temperature is equal to $\varphi(r,\theta,z)$, and the surface and base of the cylinder are maintained at a temperature equal to zero.

The problem thus reduces to solving the equation

$$\frac{\partial u}{\partial t}=a^2\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial\theta^2}+\frac{\partial^2 u}{\partial z^2}\right) \quad (1.1)$$

under boundary conditions

$$u\big|_{z=-h}=u\big|_{z=h}=0,\quad u\big|_{r=R}=0 \quad (1.2)$$

and under the initial condition

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$$u\big|_{t=0} = \varphi(r,\theta,z). \qquad (1.3)$$

By applying the Fourier method and defining the constants introduced by this method through the boundary conditions, we obtain the following particular solutions of equation (1.1):

$$e^{-a^2\left(\lambda^2+\frac{m^2\pi^2}{4h^2}\right)t} J_n(\lambda r)\sin\frac{m\pi}{2h}(z+h)(A\cos n\theta+\beta\sin n\theta). \qquad (1.4)$$

Here, the positive integers λ are denoted by , as required by the second of the boundary conditions (1.2); the constant $m$is related to the roots of the equation

$$J_n(\mu)=0 \qquad (1.5)$$

equality

$$\lambda = \frac{\mu}{R}. \qquad (1.6)$$

This is the requirement of the third of the boundary conditions (1.2). As for the number n, , it must be an integer, since the temperature of the cylinder is a periodic function of the angle θ with a period equal to 2π.

Taking now the sum of all solutions of the form (1.4), distributed over all $n = 0, 1, 2, 3, \ldots, m = 1, 2, 3, \ldots$ and for all positive roots $\mu_{n1}, \mu_{n2}, \mu_{n3}, \ldots$ equations (1.5), we obtain a solution to the problem in the form of a series:

$$u = \sum_{k=1}\sum_{m=1}\sum_{n=0} e^{-a^2\left(\frac{\mu_{nk}^2}{R^2}+\frac{m^2\pi^2}{4h^2}\right)t} J_n\left(\frac{\mu_{nk} r}{R}\right) \times$$

$$\times \sin\frac{mp}{2h}(z+h)A_{kmn}\cos n\theta+\beta_{kmn}\sin n\theta), (1.7)$$

in which it remains to determine the coefficients A_{kmn} and β_{kmn}.

For this purpose, we put $t=0$ in the expansion (1.7); then, taking into account the initial condition (3), we obtain

$$\varphi(r,\theta,z) = \sum_{k=1}\sum_{m=1}\sum_{n=0} J_n\left(\frac{\mu_{nk} r}{R}\right)\sin\frac{m\pi}{2h}(z+h)\times$$

$$\times(A_{kmn}\cos n\theta+\beta_{kmn}\sin n\theta). (1.8)$$

Since the right side of equality (1.8) is the expansion of the function $\varphi(r,\theta,z)$ in a Fourier series in $\cos n\ \theta$ and $\sin n\ \theta$, the coefficients of these trigonometric functions are determined by known formulas. Thus, we will have

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$$\frac{1}{2\pi}\int_0^{2\pi}\varphi(r,\theta,z)d\theta=\sum_{k=1}\sum_{m=1}A_{km0}\sin\frac{m\pi}{2h}(z+h)\;J_0\left(\frac{\mu_{0k}r}{R}\right),$$

$$\frac{1}{\pi}\int_0^{2\pi}\varphi(r,\theta,z)\cos n\theta d\theta=\sum_{k=1}\sum_{m=1}A_{kmn}\sin\frac{m\pi}{2h}(z+h)\;J_n\left(\frac{\mu_{nk}r}{R}\right),$$

$$\frac{1}{\pi}\int_0^{2\pi}\varphi(r,\theta,z)\sin n\theta d\theta=\sum_{k=1}\sum_{m=1}B_{kmn}\sin\frac{m\pi}{2h}(z+h)\;J_n\left(\frac{\mu_{nk}r}{R}\right).$$

Each of these equalities represents an expansion of a function considered as a function of r, in a series of Bessel functions. The coefficients of such expansions are determined by the formula:

$$\sum_{m=1}A_{km0}\sin\frac{m\pi}{2h}(z+h)=$$

$$=\frac{1}{\pi R^2J_1^2(\mu_{0k})}\int_0^R\int_0^{2\pi}r\varphi(r,\theta,z)J_0\left(\frac{\mu_{0k}r}{R}\right)drd\theta, \qquad (1.9)$$

$$\sum_{m=1}A_{kmn}\sin\frac{m\pi}{2h}(z+h)=$$

$$=\frac{2}{\pi R^2J_{n+1}^2(\mu_{nk})}\int_0^R\int_0^{2\pi}r\varphi(r,\theta,z)J_n\left(\frac{\mu_{nk}r}{R}\right)\cos n\theta drd\theta,$$

$$(1.10)\quad \sum_{m=1}B_{kmn}\sin\frac{m\pi}{2h}(z+h)=$$

$$=\frac{2}{\pi R^2J_{n+1}^2(\mu_{nk})}\int_0^R\int_0^{2\pi}r\varphi(r,\theta,z)J_n\left(\frac{\mu_{nk}r}{R}\right)\sin n\theta drd\theta, (1.11)$$

Since the functions $\sin\frac{m\pi}{2h}(z+h)$ $(m=1,\ 2,\ 3,\ .\ .\ .)$ form an orthogonal system of functions on the segment $[-h,h]$, then, using the usual method, we find that in expansions (1.9), (1.10) and (1.11) the coefficients A_{kmn}, B_{kmn}, are determined by formulas

$$A_{km0}=\frac{1}{\pi R^2hJ_1^2(\mu_{0k})}\int_0^R\int_0^{2\pi}\int_{-h}^{h}r\varphi(r,\theta,z)J_0\left(\frac{\mu_{0k}r}{R}\right)\sin\frac{m\pi}{2h}(z+h)drd\theta dz,$$

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$$A_{kmn}=\frac{2}{\pi R^2 h J_{n+1}^2\left(\mu_{nk}\right)}\int_0^R\int_0^{2\pi}\int_{-h}^{h} r\varphi(r,\theta,z)J_n\left(\frac{\mu_{nk}r}{R}\right)\cos n\theta\sin\frac{m\pi}{2h}(z+h)\,drd\theta dz,$$

$$B_{kmn}=\frac{2}{\pi R^2 h J_{n+1}^2\left(\mu_{nk}\right)}\int_0^R\int_0^{2\pi}\int_{-h}^{h} r\varphi(r,\theta,z)J_n\left(\frac{\mu_{nk}r}{R}\right)\cos n\theta\sin\frac{m\pi}{2h}(z+h)\,drd\theta dz$$

Substituting these values of the coefficients into the series (1.7), we obtain the final solution of problem (1.1) – (1.3).

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