INTERNATIONAL CONFERENCE **Dehli, India, 2026**
https://eijmr.org/conferences/

DIVISIBILITY RULES FOR 13: MATHEMATICAL PROOFS AND PRACTICAL ALGORITHMS

[1]Mahamadiyeva Aziza Sherali qizi, [2]Tursinova Sokhiba Ravshon qizi, [2]Khojakulova Nozima Abduhamid qizi
[1]Teacher of Termez State Pedagogical Institute
[2]Students of Termez State Pedagogical Institute

Abstract: The integer 13, being a prime number, presents unique challenges in mental arithmetic and computational efficiency compared to composite divisors. This article explores the mathematical foundations of divisibility rules for 13, focusing on the method of osculation and modular congruences. We provide rigorous proofs for common algorithms and evaluate their practical application in number theory and pedagogy.

Key words: Divisibility rules, divisibility by 13, mathematical proofs, practical algorithms, number theory, modular arithmetic, osculation method.

Introduction. Divisibility rules are shorthand methods used to determine whether a given integer is divisible by a fixed divisor without performing full long division. While rules for 2, 5, and 10 are intuitive, the rule for 13 requires a deeper understanding of linear Diophantine equations and the properties of the number 10 relative to the modulus 13.

The Primary Algorithm: The "Add 4 Times" Rule

The most common practical algorithm for 13 is as follows:

- Take the last digit of the number.
- Multiply it by 4.
- Add the result to the remaining part of the number.
- If the result is divisible by 13, the original number is also divisible.

Mathematical Proof

To prove this, let the integer N be represented in the form:

$N = 10a + b$

where b is the units digit ($0 \leq b \leq 9$) and a is the quotient floor(N/10).

We want to show that $N \equiv 0 \pmod{13}$ if and only if $a + 4b \equiv 0 \pmod{13}$.

Proof:

1. Consider the expression a + 4b. Multiply by 10: 10(a + 4b) = 10a + 40b.

2. Substitute 10a from N = 10a + b => 10a = N - b.

3. $10(a + 4b) = (N - b) + 40b = N + 39b$.

4. Since 39 is a multiple of 13 ($13 \times 3 = 39$), we have $39b \equiv 0 \pmod{13}$.

5. Therefore, $10(a + 4b) \equiv N \pmod{13}$. Since $\gcd(10, 13) = 1$, the proof is complete.

Alternative Algorithms. The Truncation Method (Subtraction)

One can subtract 9 times the last digit from the rest: $a - 9b \equiv 0 \pmod{13}$. This stems from the fact that $-90 \equiv 1 \pmod{13}$.

Large Number Rule (The 1001 Property)

For large numbers, utilize $7 \times 11 \times 13 = 1001$. Group digits in sets of three and form an alternating sum.

Summary Table

Method	**Formula**	**Use Case**
Osculation	a + 4b	Mental math (3-5 digits)
Subtraction	a - 9b	Theoretical calculations
Alternating Sum	G1 - G2 + G3	Large numbers (> 6 digits)

Conclusion. The divisibility rules for 13 are rooted in the cyclic nature of modular arithmetic. The (a + 4b) method remains the most computationally efficient for human use, while the 1001-property is superior for large-scale manual verification.

References:

1. G.H. Hardy and E.M. Wright, *"An Introduction to the Theory of Numbers"*. Oxford University Press.
2. D.M. Burton, *"Elementary Number Theory"*. McGraw-Hill Education.
3. T.H. Cormen, et al., *"Introduction to Algorithms"*. MIT Press.
4. M. Mirzaahmedov, Sh. Ismoilov, A. Amanov, *"Matematika: Umumiy o'rta ta'lim maktablarining 6-sinfi uchun darslik"*.
5. A.S. Vorobyov, *"Priznaki delimosti"* (Bo'linish alomatlari). Moskva, Nauka.