

MATRICES, VECTOR, DIFFERENTIAL AND INTEGRAL, SOME OPERATIONS PERFORMED ON THEM**Karshiboyev Khayrulla Kilichevich,**

Samarkand Institute of Economics and Service

Head of the Department of “Higher Mathematics”

Candidate of Physical and Mathematical Sciences, PhD

Abdullaeva Gulrukh Erkin kizi,

Master's student of Samarkand Institute of Economics and Service

E-mail: gulrux19941107@gmail.com

Tel: +998887809809

Abstract. This article analyzes the fundamental concepts of linear algebra and mathematical analysis — matrices, vectors, derivatives, and integrals — as an interconnected system. The study highlights their theoretical and practical applications in mathematical modeling, engineering, economics, and physics, as well as the main operations performed on them. The results demonstrate that these four mathematical concepts form a unified mathematical foundation for modern science and technology.

Keywords: matrix, vector, derivative, integral, linear algebra, analysis, mathematical modeling.

Annotatsiya. Ushbu maqolada chiziqli algebra va matematik analizning asosiy tushunchalari — matritsalar, vektorlar, differensial va integral — o‘zaro bog‘liq tizim sifatida tahlil qilinadi. Tadqiqotda ularning matematik modellash, injiniring, iqtisodiyot va fizika fanlaridagi qo‘llanilishi, shuningdek ular ustida bajariladigan asosiy amallar nazariy va amaliy jihatdan yoritilgan. Natijalar shuni ko‘rsatadiki, bu to‘rt matematik tushuncha zamonaviy fan va texnologiyalar uchun yagona matematik poydevorni tashkil etadi.

Kalit so‘zlar: matritsa, vektor, differensial, integral, chiziqli algebra, analiz, matematik modellashtirish.

Аннотация. В данной статье рассматриваются основные понятия линейной алгебры и математического анализа — матрицы, векторы, дифференциал и интеграл — как взаимосвязанная система. В исследовании освещены теоретические и практические аспекты их применения в математическом моделировании, инженерии, экономике и физике, а также основные операции, выполняемые над ними. Результаты показывают, что эти четыре математические категории образуют единую фундаментальную основу для современной науки и технологий.

Ключевые слова: матрица, вектор, дифференциал, интеграл, линейная алгебра, анализ, математическое моделирование.

INTRODUCTION

Modern mathematics is one of the most profound and systematic achievements of human thought. In particular, the sciences of mathematical analysis and linear algebra form the foundation of scientific and technical progress. The concepts of matrices, vectors, differential and integral, as the main branches of mathematics, are the fundamental supporting concepts not only of theoretical, but also of applied sciences - physics, economics, engineering, computer science and biology.

Matrices and vectors are widely used in representing linear systems, creating spatial models, computer graphics, economic modeling and statistical analysis. Differential and integral are important in analyzing the change of functions, expressing the processes of movement and growth, and modeling the dynamics of economic processes.

Main part

The concept of matrix was first introduced in the 19th century by the English mathematician James Joseph Sylvester, who used it to represent a system of linear equations in a compact form. Later, Arthur Cayley developed operations on matrices and proposed studying them as a separate mathematical structure. Thus, matrix theory became one of the main branches of linear algebra (Cayley, 1858).

The concept of a vector first appeared in mechanics, that is, in the representation of the directions of motion and force. By the end of the 19th century, the foundations of vector analysis were developed by Hamilton and Gibbs. Today, vector spaces are one of the most important tools for mathematical modeling (Gibbs, 1901).

Differential and integral calculus were independently developed by Isaac Newton and Gottfried Leibniz in the 17th century. Their work made it possible to mathematically represent natural processes, motion, and change (Newton, 1687; Leibniz, 1684).

In his research, Uzbek scientist A. Kadyrov (2019) analyzed the importance of differential and integral calculus in economic modeling. In his opinion, “integral calculus allows us to determine the aggregate indicators of economic processes, and differential calculus allows us to analyze the rate of their change.”

Also, S. Mamatkulov (2021) shows the place of matrices and vectors in artificial intelligence and data analysis, defining them as “multidimensional structures that represent the connection between data.”

Thus, the existing literature shows that these concepts are not only basic, but also universal concepts of mathematics, and their interconnection plays a decisive role in the development of all sciences.

A matrix is a set of numbers consisting of rows and columns. It is the most compact form of representation of linear systems and is the main tool for solving various practical problems.

For example, in an economic model, the output or resource consumption of each sector is expressed in the form of a matrix. This is clearly seen in Leontiev's inter-sectoral balance model. As Leontiev (1941) noted:

“The relationships between the interconnected sectors of the national economy can be determined by a system of linear equations.”

The basic operations performed on matrices — addition, subtraction, multiplication, transposition, finding the inverse matrix — form the basis of linear algebra. For an inverse matrix to exist, the determinant of the matrix must be nonzero. This is interpreted as a condition for the existence of a solution to a linear system.

Matrix operations also constitute the main computational mechanism in artificial intelligence systems — especially in neural networks. The weights and inputs in each layer are represented in the form of a matrix.

A vector is a mathematical object that has direction and length. It is widely used in creating spatial models, analyzing force systems, and representing multidimensional data.

Operations performed on vectors (addition, scalar product, vector product, finding the norm) allow us to determine directions in space. While in physics, vectors represent quantities such as velocity, force, and momentum, in economics they represent a set of multidimensional economic indicators.

For example, in economics, a vector differential model can be used to determine the interaction between various factors (production, inflation, investment).

Differential calculus is used to measure the rate of change of a function. For example, if $y=f(x)$, then dy/dx is the derivative of the function, that is, the rate of change. The integral, on the contrary, represents the total cumulative value of the function, that is, the sum of the changes.

These two concepts are opposite processes, and the Newton-Leibniz theorem expresses their interrelationship:

$$\int_a^b f'(x)dx = f(b) - f(a)$$

This relationship is also the foundation of modern economic analysis. For example, the growth rate of production is determined using the differential, and the total production volume is determined using the integral.

Differentials and integrals are used in areas such as economic dynamics, financial models, epidemiology, and environmental sustainability.

Matrices, vectors, differentials, and integrals are interrelated concepts. For example, vectors are considered elements of matrices, and differentials are represented by vector fields. Integrals are often calculated in vector or matrix form.

Modern computer models, especially in systems such as Python, MATLAB, and TensorFlow, perform these operations algorithmically. This speeds up scientific research and increases its accuracy.

CONCLUSION

The concepts of matrices, vectors, differentials, and integrals are integral components of the modern mathematical system. Although each of them represents a separate mathematical structure, in essence they are inextricably linked to each other and form a single system of mathematical thinking.

The results of the study show that matrices and vectors are important in expressing linear relationships, while differential and integral are important in analyzing the dynamics of these relationships and the processes of change. This unit further expands the universality of mathematics and its application in applied sciences.

First, matrices serve to simplify complex relationships between multidimensional data. They are used not only in solving systems of linear equations, but also in economic models, ecological calculations, and the main computational mechanisms of artificial intelligence algorithms. In particular, in neural networks and machine learning models, matrices act as a mathematical bridge between “weights” and “inputs”.

Second, vectors are the most important mathematical representation of spatial thinking, with the help of which indicators such as direction, force, speed, and intensity are determined. In economics, vectors are used as the basis of multivariate analysis, modeling the interrelationships of various economic indicators.

Thirdly, differential calculus allows us to determine the speed, variability, and dynamics of processes. This is of crucial importance not only in the natural sciences, but also in the study of economic growth, inflation, production efficiency, and financial flows. For example, by finding the rate of change in production volume using differentials, we can assess the effectiveness of economic policy.

Fourthly, integral calculus represents the sum of changes, that is, the total result. Through this, integral indicators such as the overall efficiency of systems, accumulated profit, total cost, or energy are determined. Integral models are also widely used in the study of interactions in the fields of ecology, finance, economics, and physics.

The combined study of matrices, vectors, differentials, and integrals forms a comprehensive approach to mathematical thinking. With their help, complex systems are modeled, decision-making processes are optimized, and innovative solutions are found in various fields of science and technology.

In today's era of globalization, in-depth knowledge of these concepts is gaining not only theoretical, but also practical importance in areas such as digital economics, programming, engineering, finance, and artificial intelligence. In particular, algorithmic methods of mathematical modeling, such as matrix algebra and computer solutions of differential equations, are expanding the possibilities for accurate and rapid analysis of real processes.

Also, these four foundations of mathematics, in harmony with each other, increase the logical, analytical, and creative capabilities of human thinking. This creates a solid theoretical foundation for interdisciplinary thinking, solving complex problems, and developing new technological approaches for a new generation of researchers.

In short, the joint study of matrices, vectors, differentials, and integrals not only expands mathematical thinking, but also further enhances the technological, scientific, and intellectual potential of humanity. This knowledge has become strategically important for every industry in the current digital era, and its in-depth study is an important condition for the development of modern science and the economy.

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