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# THE INDEPENDENCE OF EVENTS IN PROBABILITY THEORY

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**Abstract:** The concepts of independence and conditional independence of random events and random variables play a very important role in probability theory and mathematical statistics. This thesis examines the concept of independence of events in probability theory, highlighting its importance in statistical analysis and decision-making. We define events, explore their relationships, and present the mathematical criteria for determining independence. Through illustrative examples, we elucidate the practical implications of independent events, enhancing our understanding of this fundamental concept.

**Keywords:** Independence, Events, Probability Theory, Statistical Analysis, Random Experiments

## Introduction

The independence of events is a critical concept in probability theory that has significant implications in various fields such as statistics, finance, and science. An event is defined as a specific outcome or a set of outcomes resulting from a random experiment. Understanding whether events are independent—meaning the occurrence of one event does not affect the occurrence of another—is essential for accurate probability assessments and informed decision-making. This paper aims to clarify the nature of independent events and their relevance in probability theory.

A key concept in probability theory, statistics, and the theory of stochastic processes is independence. If the occurrence of one event does not, informally, influence the odds or the likelihood of the other event occurring, then two events are independent, statistically independent, or stochastically independent [1]. Likewise, if the realisation of one random variable has no effect on the probability distribution of the other, then the two variables are independent.

## Main part

It is necessary to distinguish between two concepts of independence when working with collections of more than two events. Informally, mutual independence (also known as collective independence) of events refers to the fact that each event in the collection is independent of any combination of other events, whereas pairwise independence refers to the fact that any two events in the collection are independent of one another. For groups of random variables, the idea is similar [2]. Pairwise independence does not imply mutual independence, but it does. Without more elaboration, the term "independence" typically refers to mutual independence in the mainstream literature on probability theory, statistics, and stochastic processes.

The basic units of probability theory are random occurrences, which provide scenarios with uncertain outcomes the structure they require. Randomness is a constant in our daily lives, whether we are predicting the weather, selecting lottery numbers, rolling dice, or evaluating scientific data. However, mathematics presents the abstract ideas of sample spaces and occurrences in order to thoroughly investigate these phenomena. The set of every conceivable result of a random experiment is called a sample space, and it is frequently represented by the letters S or Ω. One result from this set is called a basic outcome. Any subset of the sample space that has the potential to contain one, multiple, or all outcomes—or even none at all—is considered a random event.

For instance, the sample space for the straightforward scenario of flipping a fair coin is Ω = (Heads, Tails). The "getting Heads" event is a subset of Ω with a single result. The sample space may be the set of all real values larger than zero in more complicated situations, such as calculating a machine's lifespan, and random events may include intervals such as "machine lasts more than 5 years." Therefore, a random event is a precisely specified mathematical entity that may or may not occur during an experiment; it is not necessarily "random" in the sense that is commonly used.

Any outcome of a random experiment is called its consequence or elementary event. The set consisting of all elementary events that can occur as a result of the experiment is called the space of elementary events or the set space and is denoted by 2, and each elementary event is denoted by ω, (ω, ε, Ω).

Suppose (Ω, F, P) is a probability space, where Ω is the space of elementary events, F is the σ-algebra of subsets of Ω, and P is the probability defined in F.

Definition 1. Events A and B are said to be independent (or mutually independent) if

$P(AB)=P(A)\bullet P(B)$,

otherwise events A and B are said to be dependent.

Let A and B be independent random events,

$P(A)>0$, $P(B)>0$.

In this case, events A and B are necessarily joint.

The probability of an event A is calculated using the formula:

$P(A) = (n(A))/(n(S))$

where:

- $P(A)$ represents the probability of event A,
- $n(A)$ is the number of favorable outcomes for event A,
- $n(S)$ is the total number of possible outcomes in the sample space.

For example, when flipping a fair coin, the probability of getting heads is:

$P(Heads) = ½$

When we define the independence of two events, we specifically mentioned that the concept is introduced for a fixed probability measure P. This means that two events are independent with respect to the first probability measure, and they may not be independent with respect to the second probability measure. We will illustrate this with the help of an example involving a Bernoulli scheme, that is, a sequence of independent trials with any two outcomes 1 and 0 ("win" and "loss"). We assume that P(0) = p, where $0 \leq p \leq 1$. Then P(1) = 1-p. We conduct three independent trials and observe the following events. Let A denote the event of at most one win, and B denote the event of the same outcome. Then it is clear that A = {000,001,010,100}, B = {000,111}. Hence, it is easy to verify that the equality P(AB) = P(A)P(B) holds only in trivial cases (i.e., when p = 0 or p = 1) and p = 1/2. In all cases where the probability of winning is different, the events A and B are dependent.

Two events A and B are considered independent if the occurrence of one does not influence the occurrence of the other. This relationship can be expressed mathematically as:

$P(A \cap B) = P(A) \cdot P(B)$

Now, consider a Venn diagram depicting two events A and B.

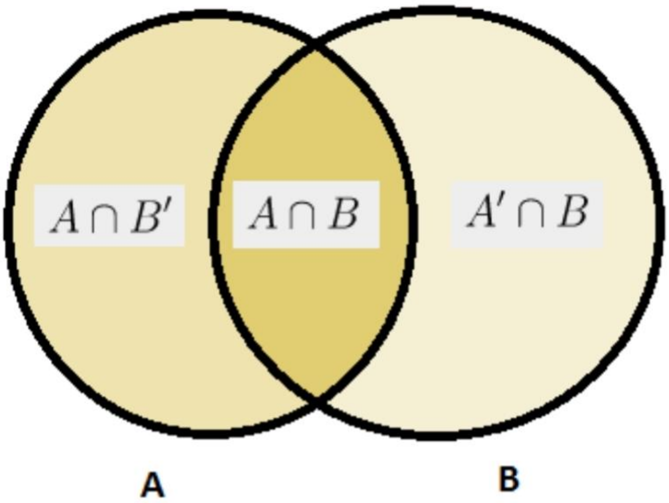


**Diagram 1. Independent events. Venn Diagram.**

If this equation holds true, then events A and B are independent.

Examples of Independent Events. Consider two independent events:

• Event A: Drawing a red card from a standard deck of cards.

• Event B: Rolling a die and getting a 4.

The probabilities are:

- $P(A) = 26/52 = ½$
- $P(B) = ⅙$

To find the joint probability of both events occurring (drawing a red card and rolling a 4), we calculate:

$P(A \cap B) = P(A) \cdot P(B) = ½ \cdot ⅙ = 1/12$

This example illustrates that the outcome of drawing a card does not affect the outcome of rolling a die.

**Theorem.** The fact that events are not related by even pairs does not imply their mutual dependence.

**Proof.** We prove the theorem for the case n = 3, that is, that the fact that three events are not related by even pairs does not imply their mutual dependence. Hence, we prove that the equality P(ABC) = P(A)P(B)P(C) does not imply the fact that events A, B, C are not related by even pairs. To prove the theorem, we give a classic example belonging to Bernstein: Suppose that there are 4 tickets with the numbers 112, 121, 211, 222 written in a box. We take one ticket at random from the box and consider the events $A_1$={the first digit of the number written on the ticket is 1). A2 ={the second digit of the number written on the ticket is 1), A3 ={the third digit of the number written on the ticket is 1). Then the equalities

$P(A_1) = P(A_2) = P(A3)= 1/2$ and $P(A_1A_2) = P(A_1A3) = P(A2A3) = 1/4$

are valid.

So, the events A1, A2, A3 are pairwise independent. However, since

$P(A1A2A3)=P(\emptyset) = 0 \neq 1/8 =P(A1)P(A2)P(A3)$,

they are mutually dependent.

The definition of random variables, conditional probability, and statistical inference are all based on random events. Random variables can be defined as functions that give numerical values to outcomes in the sample space once events have been formalised. This enables us to numerically model complex systems. Another definition of conditional probability is the calculation of the likelihood of event A given the occurrence of event B. This results in strong instruments like the Bayes theorem, which are applied in decision theory, diagnostics, and machine learning. The reasoning behind sampling theory, hypothesis testing, and prediction is based on the interaction between random events and their probability.

In mathematics, random occurrences offer a systematic and rational framework for measuring uncertainty, despite the notion of randomness connoting chaos or unpredictability. They enable scientists to create models that forecast the long-term distribution of results. The law of large numbers, which asserts that the relative frequency of an event approaches its theoretical probability as an experiment is repeated numerous times, makes this particularly clear. In a similar vein, the central limit theorem explains why the bell curve is so common in both nature and society: sums of several small, independent random occurrences tend to follow a normal distribution.

**Conclusion**

Sum up the main points, the independence of events is a fundamental principle in probability theory that simplifies the analysis of random processes. Recognizing when events are independent allows for more straightforward calculations and enhances analytical capabilities across various fields. This article provides foundational insights into independence, encouraging further exploration into its implications and applications in real-world scenarios. Understanding these principles enriches our comprehension of probability theory and equips us with essential tools for decision-making under uncertainty.

**References:**

1. Russell, Stuart; Norvig, Peter (2002). Artificial Intelligence: A Modern Approach. Prentice Hall. p. 478. ISBN 0-13-790395-2.
2. Gallager, Robert G. (2013). Stochastic Processes Theory for Applications. Cambridge University Press. ISBN 978-1-107-03975-9.
3. Park, Kun Il (2018). Fundamentals of Probability and Stochastic Processes with Applications to Communications. Springer. ISBN 978-3-319-68074-3.
4. Babayev A., Nuryyev B. ELEMENTS OF PROBABILITY THEORY: UNDERSTANDING UNCERTAINTY THROUGH MATHEMATICS // Инновационная наука. 2025. №7-2.
5. Boyan Dimitrov ON THE CONCEPTS MEANING, THEIR MEASUREMENTS AND USE IN PRACTICE OF RISK ASSESSMENT // RT&A. 2022. №SI 3 (66)