

MATHEMATICAL MODELING OF FRACTIONAL-ORDER PID (FOPID) CONTROLLERS*Sodiqov Baxtiyor Qahhor ugli**Senior teacher of department of**Automation and Control of NSUMT*

Abstract: Fractional-Order PID (FOPID) controllers extend classical PID structures by introducing two additional parameters, the fractional integral order (λ) and the fractional derivative order (μ). These degrees of freedom improve robustness, tuning flexibility, and performance in systems with nonlinearities, delays, or uncertainties. This paper presents the mathematical model of the FOPID controller, its governing equations, and simulation-based insights.

Keywords: FOPID, fractional calculus, $PI^\lambda D^\mu$ controller, mathematical modeling, robust control.

Main part of modelling FOPID controller

Fractional-Order PID (FOPID) controllers, also called $PI^\lambda D^\mu$, represent an important extension of the conventional proportional–integral–derivative (PID) control strategy. The standard PID controller is widely used in industry due to its simple structure, clear physical meaning, and ease of implementation. However, its performance can deteriorate in the presence of system nonlinearities, large delays, parameter uncertainties, and time-varying dynamics. To overcome these limitations, researchers have investigated controllers based on fractional calculus, which allows differentiation and integration of non-integer orders. The FOPID controller is the most well-known representative of this class.

The general transfer function of a FOPID controller is expressed as

$$G_c(s) = K_p + K_i s^{-\lambda} + K_d s^\mu,$$

Where, K_p is the proportional gain, K_i is the integral gain, K_d is the derivative gain,

λ is the fractional order of integration, and μ is the fractional order of differentiation. When

$\lambda=1$ and $\mu=1$, the formula reduces to the standard PID controller. The introduction of two additional parameters, λ and μ , increases the flexibility of the controller and provides more freedom in tuning the system response. One of the key advantages of FOPID controllers is their ability to achieve improved robustness. Fractional orders allow smoother adjustments of the frequency response, enabling better trade-offs between fast transient performance and disturbance rejection. In many case studies, FOPID controllers have shown reduced overshoot, faster settling times, and enhanced stability margins compared to conventional PID structures. Furthermore, they can be particularly effective in controlling systems with distributed parameters, viscoelastic dynamics, or long memory effects, where integer-order models are insufficient. The implementation of FOPID controllers requires numerical approximations of the fractional operators. Common methods include the Grünwald–Letnikov definition, the Caputo derivative, and frequency-domain approximations such as Oustaloup recursive

filters. These techniques make it possible to realize fractional-order dynamics in digital controllers, which can then be deployed in real-time applications.

Conclusion

the FOPID controller represents a promising advancement in modern control engineering. By extending the classical PID structure with fractional integration and differentiation, it provides a richer design space, higher robustness, and better adaptability to complex industrial systems. Ongoing research focuses on developing efficient tuning rules, computationally efficient approximations, and applications in emerging technologies such as autonomous systems and smart grids..

LITERATURE

1. Botirov, T., Sodiqov, B., Jumabaev, E., Salahiddinov, A., & Jurayeva, R. (2025). Design of a controller with an intelligent hybrid structure. In E3S Web of Conferences (Vol. 627, p. 04007). EDP Sciences..
2. Jumaev, O. A., Mahmudov, G. B., Rashidov, G. H., & Sodiqov, B. Q. (2023, September). Microcontroller device for increasing the accuracy of PID regulators. In AIP Conference Proceedings (Vol. 2872, No. 1, p. 120051). AIP Publishing LLC.