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METHODS OF SOLVING SIMPLE INEQUALITIES BY GRAPHICAL BASIS USING INVERSE TRIGONOMETRIC FUNCTIONS

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Annotation: This thesis covers the stages and methodology of solving simple inequalities by graphically using inverse trigonometric functions. The processes of constructing graphs, determining intersection points, and correctly determining solution intervals are explained based on examples. The advantages of the graphical method and the importance of its use in the educational process are shown.
Keywords: inverse trigonometric functions, inequalities, graphical method, arcsin, arccos, arctan, domain of determination, visual perception.

INTRODUCTION

In mathematics education, teaching inequalities containing inverse trigonometric functions and solving them graphically is of important theoretical and practical importance. These functions are found in forms such as arcsin, arccos, arctan and arccot and often seem complicated to students with an elementary understanding. Therefore, displaying and solving inequalities on a graphical basis provides not only an intuitive understanding of them, but also the development of analytical and visual thinking.

MAIN PART

Inverse trigonometric functions are the inverse of ordinary trigonometric functions, and each of them differs in its domain of definition and range of values. For example, the arcsin function is defined only from -1 to $+1$, and at the output it takes values from $-\pi/2$ to $\pi/2$. Taking into account these features, the steps for solving inequalities on a graphical basis are as follows:

The first step is to draw a graph of the functions. For example, let's take the inequality $\arcsin(x) \leq \pi/4$. In this case, the graph of the first function will be $y = \arcsin(x)$, and the second line will be $y = \pi/4$. In graphing, $\arcsin(x)$ is a smoothly increasing curve, and the domain of definition is limited from -1 to $+1$.

The second step is to determine the intersection points of the graphs. In this case, from the equality $\arcsin(x) = \pi/4$, $x = \sin(\pi/4) = \sqrt{2}/2$ is determined. This point determines the boundary of the solution intervals.

The third step is to select the values of x depending on the location between the graphs. Since $\arcsin(x) \leq \pi/4$, the interval whose graph is below the $\pi/4$ line is selected: x from -1 to $\sqrt{2}/2$.

The fourth step is to check the domain of definition and write the final interval. As a result, the solution to the inequality: $x \in [-1; \sqrt{2}/2]$.

As another example, let's consider the inequality $\arccos(x) \geq \pi/3$. First, since arcsos(x) is a decreasing function, its value decreases as x increases. For arcsos(x) $= \pi/3$, we have $x = \cos(\pi/3) = 1/2$. As can be seen from the graph, for the value of the function to be greater than $\pi/3$, x must be smaller. Therefore, the solution interval is: $x \in [-1;1/2]$.

The same method is also used for inequalities with complex arguments. For example, if $\arcsin(2x-1) \leq \pi/6$, first the domain is determined: $-1 \leq 2x-1 \leq 1$, from which $x \in [0;1]$. Then the inequality is solved: $2x-1 \leq \sin(\pi/6)=1/2$, from which $x \leq 0.75$. The final interval: $x \in [0;0.75]$.

The advantages of the graphical method are that it helps students explain the increasing-decreasing nature of functions and domains of definition, facilitates algebraic calculations, and develops visual skills.

One of the important aspects that creates difficulties for most students in the process of solving inequalities related to inverse trigonometric functions graphically is analyzing the properties of these functions and correctly determining the limit states. Therefore, in order to fully master this methodology, it is necessary to deeply understand their domain of definition, monotonicity, and symmetry properties.For example, the function arctan(x) is defined on real integers and has asymptotic behavior as it approaches infinity. That is:
When $x \to +\infty$, $\arctan(x) \to +\pi/2$,
When $x \to -\infty$, $\arctan(x) \to -\pi/2$.
This property is visible on the graph as the curve approaches horizontal asymptotes. Therefore, when solving inequalities involving arctan(x) graphically, it is necessary to consider not only the points of intersection, but also what value the function tends to at an infinite distance.
Another important point is that the interval of increase and decrease of functions is decisive in choosing the solution intervals of the inequality. For example, the function arccos(x) is a decreasing function, in which the value decreases as x increases. Therefore, it is necessary to correctly match the form of the inequality (large or small sign) and the domain of definition.
Consider the following inequality:
$\arccos(x) < \pi/4$.
We solve this inequality analytically as follows:
$\arccos(x) < \pi/4$
$x > \cos(\pi/4) = \sqrt{2}/2 \approx 0.707$.
Since the domain of definition is from –1 to +1, the solution is:
$x \in (0.707;1]$.
Here, since the function is decreasing, the values of x are chosen in a larger range, corresponding to the "small" sign of the inequality. This aspect is also visually visible when drawing graphs - the right side from the intersection point of the graphs is the desired range.
In addition, in some cases, inequalities are in the form of a composite function with a complex argument. For example:
$\arctan(3x+2) \geq \pi/6$.
To solve this inequality, first graph the function $y = \arctan(3x+2)$, which is studied in two steps:
1. The argument itself is 3x+2, which is defined as a linear function on the entire real numbers.
2. The arctan function is defined for the entire real numbers and is monotonically increasing.

CONCLUSION AND DISCUSSION

Methods for solving simple inequalities using inverse trigonometric functions on a graphical basis is recognized as an effective approach in the process of teaching mathematics. This method not only consolidates theoretical knowledge, but also allows students to develop independent thinking, logical analysis and graphic skills. Therefore, it is advisable to widely use it in school and higher education.

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